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OPTIMIZATION OF ECONOMIC PROCESSES BASED ON MATHEMATICAL PROGRAMMING METHODS

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Annotation. *The article discusses the methods of mathematical programming, namely the method of Lagrange multipliers, which is used to optimize various economic processes. The presented method allows you to move from conditional optimization to unconditional and, accordingly, expand the arsenal of available means for solving this problem. The example considers the determination of the optimal number of purchases with a limited resource.*

Keywords: *Lagrange multiplier method, optimization, allocation of limited resources, conditional extremum, partial derivative, local solution, global solution.*

In the course of their activities, manufacturing companies are faced with the need to use various resources, so the organization of procurement has a significant impact on the efficiency of its work. In a competitive environment and the ability to choose from a variety of offers, it becomes necessary to make a decision regarding such procurement characteristics as the formed product range, the list of suppliers, etc. At the same time, the company's financial resources are limited, and therefore there is a need for their rational use. Thus, the problem of purchasing optimization arises, i.e., choosing the best among the existing options, which would maximize the utility of purchases with given resources. To solve this kind of problems, there are a number of models and methods, let's consider some work in the field of company resource management.

Decision making in the economy plays a key role. There are various approaches to decision-making of economic processes. Making global decisions in the economy and other sectors of the economy using only personal experience is ineffective. The complex nature of the market economy imposes requirements on the rationale for decision-making. The formulation of the decision-making problem using modern methods of economic and mathematical modeling satisfies the above requirements.

The market economy is largely based on mathematical modeling of economic processes, and the mathematical model compiled in them delves into the basic laws of the process.

With the improvement of technology, the economy faces the task of the most appropriate allocation of production resources. If non-linear programming methods are used to solve the problem, i.e., method of Lagrange multipliers, then the extremum is sought not on the entire domain of definition, but only on a set that satisfies a certain condition.

Let us consider in more detail the Lagrange method, as it is more often used in theoretical problems of similar subjects. By itself, the economic meaning of the absolute value of the Lagrange multiplier is not very simple and is directly related to the context of the task. Let's demonstrate it on the example of the problem of production optimization.

The manufacturer's goal is to maximize output through the efficient allocation of limited resources.

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Formulation of the problem. Let production be described by the production function $L\left(\bar{x}\right)$. However, the company is able to constantly change both the amount of capital spent and the amount of labor used.

The maximum production volume in the presence of a restriction, $L\left(\bar{x}\right)$ can only be achieved through the optimal choice of values $\bar{x}=\left(x_1, x_2, ..., x_n\right)$. Consequently, the regulation of production in the long run is carried out by choosing the right set of consumed resources $\bar{x}=\left(x_1, x_2, ..., x_n\right)$, when certain conditions are imposed on the unknowns.

Based on a mathematical model, the problem of producing a manufacturer's product in the long run describes the solution of finding a set of factors $x_j \geq 0, \left(j=\overline{1,n}\right)$, which, on the one hand, satisfy the system of restrictions $g\left(x_1, x_2, ..., x_n\right)=b_i$, and on the other hand, one of these factors sets the objective function $L\left(\bar{x}\right)$ extreme value:

$$L\left(\bar{x}\right) \to \max\left(\min\right)$$

$$g_i\left(x_1, x_2, ..., x_n\right)=b_i$$

$$x_j \geq 0, \quad \left(j=\overline{1,n},\ i=\overline{1,m}\right)$$

This system reflects the above optimization problem and is a particular case of the nonlinear programming problem. It can be solved by searching for extreme values of variables in the specified area.

The method of Lagrange multipliers is used to solve the problem. The essence of the method lies in the fact that it allows you to represent the problem for the conditional extremum of the function $L\left(\bar{x}\right)$, as a problem for the unconditional extremum of the function $F\left(\bar{x}, \bar{\Lambda}\right)$ где $\bar{\Lambda}=\left(\lambda_1, \lambda_2, ..., \lambda_n\right)$, containing an additional variable λ- Lagrange multiplier. Function $F\left(\bar{x}, \bar{\Lambda}\right)$, Let us define it in such a way that, fulfilling the condition $g_i\left(x_1, x_2, ..., x_n\right)=b_i$, it did not differ from the function $L\left(\bar{x}\right)$

$$F\left(\bar{x}, \bar{\Lambda}\right)=L\left(\bar{x}\right)+\sum_{i=1}^{m}\lambda_i\left(g_i\left(\bar{x}\right)-b_i\right).$$

A necessary condition for the extremum of this function is the equality to zero of the partial derivatives of $F\left(\bar{x}, \bar{\Lambda}\right)$ by independent variables $\bar{x}, \bar{\Lambda}$:

$$\begin{cases} \dfrac{\partial F\left(\bar{x}, \bar{\Lambda}\right)}{\partial x_j}=\dfrac{\partial L\left(\bar{x}\right)}{\partial x_j}+\sum_{i=1}^{m}\lambda_i\dfrac{\partial g_i\left(\bar{x}\right)}{\partial x_j}=0, & j=\overline{1,n}; \\ \dfrac{\partial F\left(\bar{x}, \bar{\Lambda}\right)}{\partial \lambda_i}=g_i\left(\bar{x}\right)-b_i=0, & i=\overline{1,m}. \end{cases}$$

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Having solved the system, we obtain a set of points at which the objective function $L(\overline{x})$ may be extreme. That is, after solving this system of equations, we will find the values and optimal volumes of products, in our case $\overline{x} = (x_1, x_2, ..., x_n)$. Since the method allows us to identify the values of variables that maximize the volume of the product under a given constraint. It is for this volume of each product that the consumer will demand in the market.

As an example, consider purchasing optimization based on data from a confectionery firm. Information on four confectionery products is presented in the table.

Index	Product types			
	A	*B*	*C*	*D*
Forecast demand, kg	12	18	8	22
Purchase price, rub./kg	115	92	60	30

The initial data of the procurement optimization problem: the procurement budget is equal to $4500.

Based on the data, we will compose a system of equations. Now the task takes this form:

$$\begin{cases} L(\overline{x}) = (x_1 - 12)^2 + (x_2 - 18)^2 + (x_3 - 8)^2 + (x_4 - 22)^2 \\ 115x_1 + 92x_2 + 60x_3 + 30x_4 = 4500 \\ x_j \geq 0, \left(j = \overline{1,4}\right) \end{cases}$$

Where: x_1, x_2, x_3 - number of products, kg.

To solve the problem using the Lagrange multiplier method, we will transform this conditional optimization problem into an unconditional optimization problem and form the Lagrange function itself, which the company will need to minimize. To do this, we add the constraint to the original optimized function $g(x_1, x_2, x_3) = b$, multiplied by the Lagrange multiplier:

$$F(\overline{x}, \overline{\Lambda}) = (x_1 - 12)^2 + (x_2 - 18)^2 + (x_3 - 8)^2 + (x_4 - 22)^2 + \lambda(115x_1 + 92x_2 + 60x_3 + 30x_4 - 4500)$$

Let's calculate the partial derivatives, equating the obtained values to zero:

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$$\begin{cases} \dfrac{\partial F\left(\overline{x},\overline{\Lambda}\right)}{\partial x_1} = 2x_1 - 24 + 115\lambda = 0 \\ \dfrac{\partial F\left(\overline{x},\overline{\Lambda}\right)}{\partial x_2} = 2x_2 - 36 + 92\lambda = 0 \\ \dfrac{\partial F\left(\overline{x},\overline{\Lambda}\right)}{\partial x_3} = 2x_3 - 16 + 60\lambda = 0 \\ \dfrac{\partial F\left(\overline{x},\overline{\Lambda}\right)}{\partial x_4} = 2x_4 - 44 + 30\lambda = 0 \\ \dfrac{\partial F\left(\overline{x},\overline{\Lambda}\right)}{\partial \lambda} = 115x_1 + 92x_2 + 60x_3 + 30x_4 - 4500 = 0 \end{cases}$$

Now it remains for us to solve this system of equations. From where we get that:

$$\left(x_1, x_2, x_3, x_4\right) = \left(13,43;\ 19,44;\ 8,73;\ 22,37\right);\ \lambda = -0,025\cdot$$

Let's try to determine the type of extremum at the found point using a sufficient condition. It states that the type of extremum is determined by the sign of the second differential of the Lagrange function

Construct the total differential of the second-order Lagrange function and determine its sign at each of the stationary points. Stationary point $\left(x_1,\ x_2,\ ...,x_n\right)$ is a local maximum point if $d^2F<0$, and minimum if $d^2F>0$.

The total second-order differential of the Lagrange function

$F\left(\overline{x},\overline{\Lambda}\right)$ has the form

$$d^2F\left(\overline{x},\overline{\Lambda}\right) = \sum_{j=1}^{n} \frac{\partial^2 L\left(\overline{x}\right)}{\partial^2 x_j^2} dx_j^2 + 2\sum_{i\neq j}^{n} \frac{\partial^2 L\left(\overline{x}\right)}{\partial x_i \partial x_j} dx_i dx_j\,.$$

If at a stationary point $d^2F<0$, then the function reaches its maximum if $d^2F>0$ – then the minimum; if d^2F turns out to be alternating, then additional steps will be required.

Let's find partial derivatives of the 2nd order:

$$\frac{\partial F^2}{\partial x_1^2} = \left(2x_1 - 24 + 115\lambda\right)'_{x_1} = 2,\quad \frac{\partial F^2}{\partial x_1 \partial x_2} = \left(2x_1 - 24 + 115\lambda\right)'_{x_2} = 0,\quad \frac{\partial F^2}{\partial x_1 \partial x_3} = \left(2x_1 - 24 + 115\lambda\right)'_{x_3} = 0,$$

$$\frac{\partial F^2}{\partial x_1 \partial x_4} = \left(2x_1 - 24 + 115\lambda\right)'_{x_4} = 0,\quad \frac{\partial F^2}{\partial x_2^2} = \left(2x_2 - 36 + 92\lambda\right)'_{x_2} = 2,\quad \frac{\partial F^2}{\partial x_2 \partial x_3} = \left(2x_2 - 36 + 92\lambda\right)'_{x_3} = 0,$$

$$\frac{\partial F^2}{\partial x_2 \partial x_4} = \left(2x_2 - 36 + 92\lambda\right)'_{x_4} = 0,\quad \frac{\partial F^2}{\partial x_3^2} = \left(2x_3 - 16 + 60\lambda\right)'_{x_2} = 2,\quad \frac{\partial F^2}{\partial x_3 \partial x_4} = \left(2x_3 - 16 + 60\lambda\right)'_{x_4} = 0,$$

$$\frac{\partial F^2}{\partial x_4^2} = \left(2x_4 - 44 + 30\lambda\right)'_{x_4} = 2.$$

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As applied to a given problem, the total second-order differential of the Lagrange function takes the form

$$d^2F\left(\overline{x},\overline{\Lambda}\right)=\frac{\partial^2F\left(\overline{x},\overline{\Lambda}\right)}{\partial^2x_1^2}dx_1^2+\frac{\partial^2F\left(\overline{x},\overline{\Lambda}\right)}{\partial^2x_2^2}dx_2^2+\frac{\partial^2F\left(\overline{x},\overline{\Lambda}\right)}{\partial^2x_3^2}dx_3^2+\frac{\partial^2F\left(\overline{x},\overline{\Lambda}\right)}{\partial^2x_4^2}dx_1^2+$$

$$+2\frac{\partial^2F\left(\overline{x},\overline{\Lambda}\right)}{\partial x_1\partial x_2}dx_1dx_2+2\frac{\partial^2F\left(\overline{x},\overline{\Lambda}\right)}{\partial x_1\partial x_3}dx_1dx_3+2\frac{\partial^2F\left(\overline{x},\overline{\Lambda}\right)}{\partial x_1\partial x_4}dx_1dx_4+2\frac{\partial^2F\left(\overline{x},\overline{\Lambda}\right)}{\partial x_2\partial x_3}dx_2dx_3+$$

$$+2\frac{\partial^2F\left(\overline{x},\overline{\Lambda}\right)}{\partial x_2\partial x_4}dx_2dx_4+2\frac{\partial^2F\left(\overline{x},\overline{\Lambda}\right)}{\partial x_3\partial x_4}dx_3dx_4=2\cdot dx_1^2+2\cdot dx_2^2+2\cdot dx_3^2+2\cdot dx_4^2+2\cdot 0\cdot dx_1dx_2+$$

$$+2\cdot 0\cdot dx_1dx_3+2\cdot 0\cdot dx_2dx_3+2\cdot 0\cdot dx_2dx_4+2\cdot 0\cdot dx_3dx_4=2\cdot dx_1^2+2\cdot dx_2^2+2\cdot dx_3^2+2\cdot dx_4^2>0$$

Therefore, from the inequality $d^2F\left(\overline{x},\overline{\Lambda}\right)>0$ it follows that the function $L\left(\overline{x}\right)=\left(x_1-12\right)^2+\left(x_2-18\right)^2+\left(x_3-8\right)^2+\left(x_4-22\right)^2$ reaches a conditional minimum at the point $\left(x_1,\ x_2,\ ...,x_n\right)=\left(13,43;\ 19,44;\ 8,73;\ 22,37\right)$.

Thus, optimizing the purchase of these four goods A, B, C and D, the company needs to purchase 13.43 kg of product A, 19.44 kg of product B, 8.73 kg of product C and 22.37 kg of product D.

Summarizing the above, it should be noted that the economic system covers various parameters and features of production, such as distribution, exchange and consumption of material resources. The functioning of economic systems is inherently multi-criteria. Applying the methods of mathematical programming to calculate conditional extremes in economic research, we will be able to find the extreme values of many economic indicators, namely the maximum profit, minimum production costs, and also allow us to solve economic problems for optimizing production processes in general.

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