ASIA PACIFIC JOURNAL OF MARKETING & MANAGEMENT REVIEW
ISSN: 2319-2836
IMPACT FACTOR: 7.603
Vol 11, Issue 05, 2022

AN ILL-POSED PROBLEM FOR AN ABSTRACT BICALORIC EQUATION.

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Abstract: *In article the incorrect task for abstract bicaloric the equation is studied and a stability assessment according to Tikhonov is given.*

Keywords: *bicaloric, spaces, self-conjugate, linear, unlimited,*

dense,operator, theorems.

A task. It is required to find a solution to the abstract bicaloric equation

$$K_+^2 u(t) \equiv \left(\frac{d}{dt} + A\right)^2 u(t) = 0,\ 0 < t < T, \quad (1)$$

satisfying the following conditions:

$$\left.\begin{array}{l} u|_{t=l_1} = u(l_1) \\ u|_{t=l_2} = u(l_2) \end{array}\right\} \qquad (2)$$

where $u(t)$ - abstract function with values in Hilbert space H.

A - constant, positive-definite, self-adjoint, linear, unbounded with everywhere dense domain $D(A^2)$ (DCH) operator operating from H in H, and $u(l_1), u(l_2) \in H$.

The validity of the representation is proved.

$$u = u_1 + (t - l_1)u_2 .$$

Theorem. If a u_1 and u_2 are solutions of the caloric equation, then the function $u = u_1 + (t - l_1)u_2$ is a solution to equation (1) and vice versa, for each given abstract bicaloric function there are such functions u_1 and u_2 what

$$u = u_1 + (t - l_1)u_2$$

ISSN 2319-2836 (online), Published by
ASIA PACIFIC JOURNAL OF MARKETING & MANAGEMENT REVIEW.,
under Volume: 11 Issue: 05 in May-2022
https://www.gejournal.net/index.php/APJMMR

Proof. 1) If u_1 and u_2 solution of the caloric equation, that is, the solution of the bicaloric equation

$$K_+u = K_+\left[u_1 + (t-1_1)u_2\right] = K_+u_1 + u_2 + (t-1_1)\frac{du_2}{dt} + A(t-1_1)u_2 =$$
$$= u_2 + (t-1_1)\left(\frac{du_2}{dt} + Au_2\right) = u_2 + (t-1_1)\cdot K_+u_2 = u_2.$$

Because

$\dfrac{du_2}{dt} + Au_2 = 0,$ то $K_+\left(u_1 + (t-1_1)u_2\right) = u_2$ т-е $K_+u = u_2$.

Applying again the operator K_+, given that $K_+u_2 = K_+K_+u = 0;$

2) If u solution of the bicaloric equation, then there are such caloric functions u_1 , u_2 what $u = u_1 + (t-l_1)u_2$.

To prove this assertion, it suffices to establish the possibility of choice u_2.

Let's put

$$u_2 = K_+u,$$
$$u_1 = u - (t-l_1)u_2.$$

It remains to show that

$$K_+\left[u - (t-l_1)u_2\right] = 0.$$

Indeed:

$$K_+u_1 = K_+\left[u - (t-l_1)u_2\right] = K_+u - K_+(t-l_1)u_2 =$$
$$= K_+u - u_2 - (t-l_1)\cdot\frac{du_2}{dt} - A\cdot(t-l_1)u_2 =$$
$$= K_+u - u_2 - (t-l_1)\cdot\left(\frac{du_2}{dt} - Au_2\right) = K_+u - u_2 = 0,$$

from here

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$K_+u_1 = 0, \quad K_+u_2 = 0.$

The theorem is completely proven.

With the help of a view

$$u = u_1 + (t - l_1)u_2 \qquad (3)$$

The solution of problem (1) - (2) can be reduced to solving the following problems:

$$\begin{cases} K_+u_1 = 0, & (4) \\ u_1\big|_{t=l_1} = u(l_1) & (5) \end{cases}$$

and

$$\begin{cases} K_+u_2 = 0, & (6) \\ u_2\big|_{t=l_2} = u_2(l_2) & (7) \end{cases}$$

where $u_2(l_2) = \dfrac{u(l_1)}{l_2 - l_1} - \dfrac{u_1(l_2)}{l_2 - l_1}$, $\qquad u_1(l_2) = \|u(0)\|^{\frac{l_1 - l_2}{l_1}} \|u(l_1)\|^{\frac{l_2}{l_1}}$

a task (4) – (5) $0 < t < l_1$ incorrect in the classical sense, *a* $l_1 < t < T$ correctly. Problem (4) - (5) will be investigated for conditional correctness according to Tikhonov

Theorem. For any solution of problem (4) - (5), the inequality is true.

$$\|u_1(t)\| \le \|u(0)\| \frac{l_1 - t}{l_1} \cdot \|u(l_1)\|^{\frac{t}{l_1}}.$$

Proof. Consider the function [1]

$$\varphi(t) = \|u_1(t)\|^2 = (u_1, u_2).$$

Differentiating it, we get

$$\varphi'(t) = 2(u_1', u_1) = 2(Au_1, u_1)$$

$$\varphi''(t) = 2(u_1', u_1) + 2(u_1, u_1'') = 2(Au_1, Au_1) + 2(u_1, A^2 u_1).$$

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Since the operator is self-adjoint $\left(m.e. A = A^*\right)$, то $\left(u_1, A^2 u_1\right) = \left(Au_1, Au_1\right)$ and that means, $\varphi''(t) = 4\left(Au_1, Au_1\right)$.

Now consider the function

$$\psi(t) = ln\,\varphi(t)$$

Differentiating it, we have

$$\psi''(t) = \frac{1}{\varphi^2(t)}\left[\varphi''(t)\cdot\varphi'^2(t)\right] = \frac{4}{\varphi^2(t)}\left[\left(Au_1, Au_1\right)\left(u_1, u_1\right) - \left(Au_1, u_1\right)^2\right] \geq 0 \qquad (8)$$

By virtue of the well-known Bunyakovskii inequality, inequality (8) means that the function $\psi(t)$ turned concave upwards, from which it follows that the function $\psi(t)$ on the segment $\left[0, l_1\right]$ does not exceed a linear function that takes the same values at the ends of the segment as $\psi(t)$. From (8) it follows

$$\psi(t) \leq \frac{l_1 - t}{l_1}\psi(0) + \frac{t}{l_1}\psi\left(l_1\right) \qquad (9)$$

Potentiating inequality (9), we obtain

$$\varphi(t) \leq \left[\varphi(0)\right]\frac{l_1 - t}{l_1}\cdot\left[\varphi\left(l_1\right)\right]^{\frac{t}{l_1}},$$

Where $\left\|u_1(t)\right\| \leq \left\|u(0)\right\|^{\frac{l_1 - t}{l_1}}\cdot\left\|u\left(l_1\right)\right\|^{\frac{t}{l_1}}$

A task (6) – (7) $0 < t < l_2$ incorrectly, a $l_2 < t < T$ correct in the classical sense, similarly to problem (4) - (5) it can be examined for conditional correctness according to Tikhonov

Let us prove a theorem characterizing the stability estimate for the solution of the problem

(1) – (2)

Theorem. For any solution of problem (1) – (2), the inequality

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$$\|u(t)\|_H \le \|u(0)\|^{\frac{l_1-t}{l_1}} \|u(l_1)\|^{\frac{t}{l_1}} +$$

$$+(t-l_1)\begin{cases} \frac{1}{l_2-l_1}\left(\|u(l_2)\| + \|u(0)\|^{\frac{l_1-l_2}{l_1}} \|u(l_1)\|^{\frac{l_2}{l_1}}\right)^{\frac{t}{l_2}} \cdot \|u(l_1)\|^{\frac{t-l_1}{l_1}}, & l_1 < t < l_2 \\ \frac{1}{T-l_1}\left(\|u(T)\| + \|u(0)\|^{\frac{l_1-T}{l_1}} \|u(l_1)\|^{\frac{T}{l_1}}\right)^{\frac{T-t}{T}} \cdot \|u(l_2)\|^{\frac{t}{l_2}}, & l_2 \le t \le T \end{cases} \quad (10)$$

Note that inequality (10) implies the uniqueness of the solution to problem (1)–(2) and the conditional well-posedness of this problem in the class

$$\{u : \|u(0)\| \le M\}$$

This theorem is proved by the logarithmic convexity method

References:

1. **Лаврентьев М.М.** *Некорректные задачи для дифференциальных уравнений. Новосибирск: Изд-ва НГУ, 1981, 71 с.*

2. **Кабанихин. С.И.** *Обратные и некорректные задачи. Новосибирск. 2009 г.*

3. **Эгамбердиев О.М.** *Задача Коши для абстрактного поликалорического уравнения. ДАН РУз. № 6. 2011*, с. 18-20.

4. **Атахаджаев М.А, Эгамбердиев О.М.** *Задача Коши для абстрактного бикалорического уравнения. Сибирский математический журнал. Т.31, № 4, 1990.*