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INFLUENCE OF INVOLUTION ON DIFFERENTIAL EQUATIONS WITH SECOND-ORDER CONSTANT COEFFICIENTS

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Abstract: *The paper presents examples of solving simple differential equations with evolutionary properties and non-homogeneous second-order differential equations.*

Key words: *involution, simple differential equations and non-homogeneous ordinary differential equations with involution.*

DISCUSSION

As we know if the equation $f\left(f\left(x\right)\right)=x$ is true when $f:R\to R$ *is reflected, this reflection is called involution.*

In the science of differential equations, we also encounter a number of differential equations that have the property of involution. The functions as $f(x)=\sqrt[n]{1-x^n}$, $n\epsilon N$, $f(x)=\frac{\alpha x+\beta}{\mu x-\lambda}$ composes involution.

We can distinguish some of these functions and apply them to differential equations. Let us consider the differential equation of the second-order variable coefficient as follows.

$$y''\left(\frac{1}{x}\right)+ay'(x)=q(x) \qquad \mathbf{(1)}$$

Here $a=const,\ q(x)$ $-$free koef.

Theorem: (1) The second order after the equations is the integration of the simple differential equation

Proof: if we do reflection for (1) equation as $f:x\to\frac{1}{x}$, the result will be as follows:

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$$y''(x)+ay'\left(\frac{1}{x}\right)=q\left(\frac{1}{x}\right) \qquad \textbf{(2)}$$

From (2) equation we find $y'\left(\frac{1}{x}\right)=\frac{1}{a}\left(q\left(\frac{1}{x}\right)-y''(x)\right)$ and find product by x. $y''\left(\frac{1}{x}\right)=\frac{x^2}{a}\left(\frac{1}{x^2}q'\left(\frac{1}{x}\right)+y'''(x)\right)$ we put this result to the (1) equation.

$\frac{x^2}{a}\left(\frac{1}{x^2}q'\left(\frac{1}{x}\right)+y'''(x)\right)+ay'(x)=q(x)$ and when we simplify it the result will be as follows:

$$y'''(x)+\frac{a^2}{x^2}y'(x)=\frac{a}{x}\,q(x)-\frac{1}{x}q'\left(\frac{1}{x}\right) \textbf{(3)}$$

We multiply two parts of (3) by x^3 and the result will be as follows:

$$x^3y'''(x)+a^2xy'(x)=g(x)\textbf{(4)}$$

$$\text{here} \quad x^2a\,q(x)-x^2q'\left(\frac{1}{x}\right)=g(x)$$

the equation (4) come to the problem of solving simple differential equation. ■

The equation (4) is also called Eyler problem. We consider it the general solution of homogeneity. In this case, we bring the linear differential equation with a variable coefficient to the linear equation with a variable coefficient. For this, we do change as $x=e^t$ and take the following result:

From $\frac{dx}{dt}=e^t$ it comes to $\frac{dt}{dx}=e^{-t}$

$$y'_x=\frac{dy}{dx}=\frac{dy}{dt}\cdot\frac{dt}{dx}=e^{-t}\cdot y'_t$$

$$y''_{xx}=\frac{d^2y}{dx^2}=\frac{d}{dx}\left(\frac{dy}{dx}\right)=\frac{d}{dx}(e^{-t}\cdot y'_t)=e^{-t}\cdot\frac{d}{dx}\left(\frac{dy}{dt}\right)$$

$$\frac{d}{dx}\left(\frac{dy}{dt}\right)=\frac{d}{dt}\left(\frac{dy}{dx}\right)$$

and it results

$$y''_{xx}=e^{-t}\cdot\frac{d}{dt}(y'_x)=e^{-t}\cdot\frac{d}{dt}(e^{-t}\cdot y'_t)=e^{-t}\left[-e^{-t}\frac{dy}{dt}+e^{-t}\cdot\frac{d^2y}{dt^2}\right]=$$

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$$= e^{-t}\left[-e^{-t}\frac{dy}{dt} + e^{-t}\cdot\frac{d^2y}{dt^2}\right] = e^{-2t}\left[\frac{d^2y}{dt^2} - \frac{dy}{dt}\right]$$

From the above given, the following will be true:

$$y_x''' = e^{-3t}\cdot\left[\frac{d^3y}{dt^2} - 3\frac{d^2y}{dt^2} + 2\frac{dy}{dt}\right]$$

We put for the (4) equation amount of $y_x', {y''}_{xx}, y_x'''$ The result will be as follows:

$$y_t''' - 3y_t'' + 2y_t' + a^2y_t' = g(e^t) \qquad \mathbf{(5)}$$

(5) is a third-order differential equation with a constant coefficient of non-homogeneity.

The solution for (5) is as:

$$y = Y + Y_1 \qquad \mathbf{(5')}$$

Here Y is the solution for homogeneous part of the equation (5), and Y_1 is a particular solution.

Let us solve homogeneous part of the equation (5)

$$y_t''' - 3y_t'' + (a^2+2)y_t' = 0 \qquad \mathbf{(6)}$$

Here (6) we do change as $y = e^{kt}$ and find characteristic equation.

$$k^3 - 3k^2 + (a^2+2)k = 0$$

$$k_1 = 0$$

$$k_{2,3} = \frac{3 \pm \sqrt{9 - 4(a^2+2)}}{2} = \frac{3 \pm \sqrt{1-4a^2}}{2}$$

From these

$$y_1 = c_1, y_2 = e^{\frac{3t}{2}}\left(c_2 cos\frac{\sqrt{1-4a^2}t}{2} + c_3 sin\frac{\sqrt{1-4a^2}t}{2}\right) \qquad \mathbf{(7)}$$

and $x = e^t$ we put $t = lnx$ to the equation (7).

$$y_1 = c_1, y_2 = x^{\frac{3}{2}}(c_2 cos\frac{\sqrt{1-4a^2}lnx}{2} + c_3 sin\frac{\sqrt{1-4a^2}lnx}{2})$$

The solution of the homogeneous part is a combination of these lines:

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$$Y = c_1 + x^{\frac{3}{2}}(c_2 cos\frac{\sqrt{1-4a^2}lnx}{2} + c_3 sin\frac{\sqrt{1-4a^2}lnx}{2})$$

We find Y_1 according to $g(x)$

Example: Find the general solution for

$$y''\left(\frac{1}{x}\right) + y'(x) = x$$

Solution: Here we do reflection as $f: x \to \frac{1}{x}$and the result as follows:

$y''(x) + y'\left(\frac{1}{x}\right) = \frac{1}{x}$ and from this we find $y'\left(\frac{1}{x}\right) = \frac{1}{x}$ and take once production:

$$y''\left(\frac{1}{x}\right) = x^2\left(y'''(x) + \frac{1}{x^2}\right)$$

$$y''\left(\frac{1}{x}\right) = x^2 y'''(x) + 1$$

$$x^2 y'''(x) + 1 + y'(x) = x/\, x$$

$$x^3 y''' + xy'(x) = x(x-1) \qquad (\mathbf{8})$$

If we change as $(8) x = e^t$ we get

$$y'''_t - 3y''_t + 3y'_t = e^t(e^t - 1) \qquad (\mathbf{8'})$$

As $a = 1$ in (8) the solution is as follows:

$$Y = x\sqrt{x}\left[c_1 \cos\frac{\sqrt{3}}{2}lnx + c_2 \sin\frac{\sqrt{3}}{2}lnx\right) + c_3$$

Let us find a particular solution for (8)

We search a particular solution for $(8')$ as

$$y_1(t) = Ae^{2t} + Be^t$$

$$y'_1(t) = 2Ae^{2t} + Be^t$$

$$y''_1(t) = 4Ae^{2t} + Be^t$$

$$y'''_1(t) = 8Ae^{2t} + Be^t$$

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We put them to the $(8')$

$$(8A - 12A + 6A)e^{2t} + Be^t = e^{2t} - e^t$$

We equate the corresponding coefficients of the equation. The result is $A = \frac{1}{2}; B = -1$. From this it comes to

$$y_1(t) = \frac{1}{2}e^{2t} - e^t$$

The particular solution for (8):

$$Y_1 = \frac{1}{2}x^2 - x$$

So we find that the general solution of the equation derived from the above is :

$$y(x) = x\sqrt{x}(c_1 \cos\frac{\sqrt{3}}{2}lnx + c_2 \sin\frac{\sqrt{3}}{2}lnx) + c_3 + \frac{1}{2}x^2 - x$$

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