INCORRECT PROBLEM FOR AN ABSTRACT BICALORIC EQUATION

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Abstract. *This study explores solutions to an abstract bicaloric equation, integrating the concepts of a Hilbert space and a self-adjoint operator, coupled with the caloric equation. The article establishes a crucial theorem that establishes a link between solutions to the bicaloric equation and the caloric equation. It further streamlines the problem by transforming it into related equations. The focus of this investigation is primarily on assessing the correctness and stability of these solutions. The logarithmic convexity method is employed to substantiate the findings and conclusions of the study.*

Keywords: *Bicaloric equation, Hilbert space, self-adjoint operator, theorem, logarithmic convexity, stability assessment.*

In the realm of abstract mathematics, this study delves into the complexities surrounding the abstract bicaloric equation, its associated conditions, and the search for solutions.

Problem: Find a solution to the abstract bicaloric equation:

$$K_+^2 u(t) \equiv \left(\frac{d}{dt} + A\right)^2 u(t) = 0, \; 0 < t < T, \tag{1}$$

satisfying the following conditions:

$$\left.\begin{aligned} u\big|_{t=l_1} &= u(l_1) \\ u\big|_{t=l_2} &= u(l_2) \end{aligned}\right\} \tag{2}$$

Where: $u(t)$ - abstract function with values in Hilbert space H.

A - constant, positive definite, self-adjoint, linear, unbounded with dense domain of definition $D(A^2)$ (DCH) operator operating from H in H, and $u(l_1), u(l_2) \in H$.

The validity of the representation is proven.

$$u = u_1 + (t - l_1)u_2 .$$

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Theorem 1. If u_1 and u_2 there are solutions to the caloric equation, then the function $u = u_1 + (t - l_1)u_2$ is a solution to equation (1) and vice versa, for each given abstract bicaloric function u there are such functions u_1 and u_2 what

$$u = u_1 + (t - l_1)u_2$$

Proof. 1) If u_1 and u_2 solutions to the caloric equation, then u is a solution to the bicaloric equation

$$K_+u = K_+\left[u_1 + (t - 1_1)u_2\right] = K_+u_1 + u_2 + (t - 1_1)\frac{du_2}{dt} + A(t - 1_1)u_2 =$$

$$= u_2 + (t - 1_1)\left(\frac{du_2}{dt} + Au_2\right) = u_2 + (t - 1_1)\cdot K_+u_2 = u_2.$$

Because $\frac{du_2}{dt} + Au_2 = 0,$ that $K_+\left(u_1 + (t - 1_1)u_2\right) = u_2$ т-е $K_+u = u_2$.

Using the operator again K_+, considering, that $K_+u_2 = K_+K_+u = 0;$

2) If u solution of the bicaloric equation, then there are such caloric functions u_1, u_2 what $u = u_1 + (t - l_1)u_2$.

To prove this statement, it is enough to establish the possibility of choice u_2.

Let's put:

$$u_2 = K_+u,$$

$$u_1 = u - (t - l_1)u_2.$$

It remains to show that:

$$K_+\left[u - (t - l_1)u_2\right] = 0.$$

Indeed:

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$$K_+u_1 = K_+\left[u-(t-l_1)u_2\right] = K_+u - K_+(t-l_1)u_2 =$$
$$= K_+u - u_2 - (t-l_1)\cdot\frac{du_2}{dt} - A\cdot(t-l_1)u_2 =$$
$$= K_+u - u_2 - (t-l_1)\cdot\left(\frac{du_2}{dt} - Au_2\right) = K_+u - u_2 = 0,$$

from here:

$$K_+u_1 = 0,\ \ K_+u_2 = 0.$$

The theorem is completely proven.

Using the view:

$$u = u_1 + (t-l_1)u_2 \qquad (3)$$

The solution to the problems (1) – (2) can be reduced to solving the following problems:

$$\begin{cases} K+u_1=0, \\ u_1|_{t=l_1=u(l_1)}. \end{cases} \qquad (4)$$

$$\begin{cases} K+u_2=0, \\ u_2|_{t=l_2=u_2(l_2)}. \end{cases} \qquad (5)$$

Where: $u_2(l_2) = \dfrac{u(l_1)}{l_2-l_1} - \dfrac{u_1(l_2)}{l_2-l_1},$ $\qquad u_1(l_2) = \|u(0)\|^{\frac{l_1-l_2}{l_1}}\|u(l_1)\|^{\frac{l_2}{l_1}}$

tasks (4) $0<t<l_1$ incorrect in the classical sense, *a* $l_1<t<T$ correctly.

We will examine problems (4) for conditional correctness according to Tikhonov. Theorem 2. For any solution to problem (4), the inequality holds. $\|u_1(t)\| \le \|u(0)\|\dfrac{l_1-t}{l_1}\cdot\|u(l_1)\|^{\frac{t}{l_1}}$.

Task (5) $0<t<l_2$ incorrect, *a* $l_2<t<T$ in the classical sense is correct, similarly to problem (4), it can be examined for conditional correctness according to Tikhonov.

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Let us prove a theorem characterizing the stability assessment of the solution to problem (1) – (2)

Theorem 3. For any solution to problem (1) – (2) the following inequality is true: $\|u(t)\|_H \le \|u(0)\|^{\frac{l_1-t}{l_1}} \|u(l_1)\|^{\frac{t}{l_1}} +$

$$+(t-l_1)\begin{cases} \dfrac{1}{l_2-l_1}\left(\|u(l_2)\| + \|u(0)\|^{\frac{l_1-l_2}{l_1}} \|u(l_1)\|^{\frac{l_2}{l_1}}\right)^{\frac{t}{l_2}} \cdot \|u(l_1)\|^{\frac{t-l_1}{l_1}}, & l_1 < t < l_2 \\ \dfrac{1}{T-l_1}\left(\|u(T)\| + \|u(0)\|^{\frac{l_1-T}{l_1}} \|u(l_1)\|^{\frac{T}{l_1}}\right)^{\frac{T-t}{T}} \cdot \|u(l_2)\|^{\frac{t}{l_2}}, & l_2 \le t \le T \end{cases}$$

This theorem is proven by the logarithmic convexity method [1].

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