STATEMENT AND STUDY OF A BOUNDARY VALUE PROBLEM FOR A THIRD-ORDER EQUATION OF PARABOLIC-HYPERBOLIC TYPE IN A MIXED PENTAGONAL DOMAIN, WHEN THE SLOPE OF THE CHARACTERISTIC OF THE OPERATOR THE FIRST ORDER IS GREATER THAN ONE

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Annotation: *In this paper, we pose and study one boundary value problem for a third-order parabolic-hyperbolic equation of the form* $\left(a\frac{\partial}{\partial x}+b\frac{\partial}{\partial y}+c\right)(Lu)=0$ *in the pentagonal region, when the characteristic of the operator* $a\frac{\partial}{\partial x}+b\frac{\partial}{\partial y}+c$ *more than one. The unique solvability of the problem posed is proved using the method of constructing a solution.*

Key words*: Boundary value problem, parabolic-hyperbolic type, unique solvability, solution construction methods, continuous derivatives, boundary conditions, gluing conditions.*

Introduction

It is known that mixed equations of the second order of elliptic-hyperbolic type were initially studied. Fundamental research on such equations began in the 1920s by the Italian mathematician Tricomi [1] and developed by Gellerstedt [2], AV Bitsadze [3, 4], KI Babenko [5], IL Karol [6], FI Frankl [7], MM Smirnov [8], MS Salakhitdinov [9] and others.

Investigations of the equations of elliptic-parabolic and parabolic-hyperbolic types of the second order began in the 50-60s of the last century. In 1959, IM Gel'fand [10] pointed out the need for joint consideration of equations in one part of the region of parabolic, and in the other part, of hyperbolic type. He gives an example related to the movement of a gas in a channel surrounded by a porous medium: in a channel, the movement of a gas is described by a wave equation, outside it - by a diffusion equation.

Then, in the 70-80s of the twentieth century, research began on equations of the third and higher orders of parabolic-hyperbolic type. Boundary value problems for such were posed and studied for the first time by TD Djuraev [11] and his students [12], [13].

Formulation of the problem

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In this work, one boundary value problem is posed and investigated for a third-order parabolic-hyperbolic equation of the form

$\left(a\frac{\partial}{\partial x}+b\frac{\partial}{\partial y}+c\right)(Lu)=0$ in the pentagonal area G of the plane xOy, where

$$Lu=\begin{cases}u_{xx}-u_y, & (x,y)\in G_1,\\ u_{xx}-u_{yy}, & (x,y)\in G_i \quad (i=2),\end{cases}$$

$a,b,c\in R$, $\gamma=\frac{b}{a}$, $1<\gamma<+\infty$, a $G=G_1\cup G_2\cup G_3\cup J_1\cup J_2$, $G_1=\{(x,y)\in R^2: 0<x<1,\ 0<y<1\}$,

$G_2=\{(x,y)\in R^2: -1<y<0,\ -1-y<x<1+y\}$, $G_3=\{(x,y)\in R^2: -1<x<0,\ 0<y<1\}$,

$J_1=\{(x,y)\in R^2: y=0,\ -1<x<1\}$, $J_2=\{(x,y)\in R^2: x=0,\ 0<y<1\}$,

that is G_1- rectangle with vertices at points $A(0;0)$, $B(1;0)$, $B_0(1,1)$, $A_0(0,1)$; G_2- triangle with vertices at points B, $C(0,-1)$, $D(-1,0)$;

G_3- rectangle with vertices at points A, A_0, $D_0(-1,1)$, D;

J_1- open line segment with vertices at points B, D;

J_2- open line segment with vertices at points A, A_0.

Here in this work, since $\gamma=\frac{b}{a}$ and $1<\gamma<+\infty$, then without loss of generality we can assume $a>0$, $b>0$. For equation (1), the following problem is posed:

Problem 1. You want to find a function $u(x,y)$, which 1) is continuous in $\overline{G}$ and in the area $G\setminus J_1\setminus J_2$ has continuous derivatives involved in equation (1), Moreover u_x and u_y continuous in G up to part of the area boundary G specified in the boundary conditions; 2) satisfies Eq. (1) in the region $G\setminus J_1\setminus J_2$ 3) satisfies the following boundary conditions:

$$u(1,y)=\varphi_1(y),\quad 0\le y\le 1, \qquad (2)$$

$$u(-1,y)=\varphi_2(y),\quad 0\le y\le 1, \qquad (3)$$

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$$u_x(-1,y)=\varphi_3(y),\ 0\le y\le 1, \qquad (4)$$

$$u|_{BC}=\psi_1(x),\ 0\le x\le 1, \qquad (5)$$

$$u|_{DF}=\psi_2(x),\ -1\le x\le -1/2, \qquad (6)$$

$$\left.\frac{\partial u}{\partial n}\right|_{BC}=\psi_3(x),\ 0\le x\le 1, \qquad (7)$$

$$\left.\frac{\partial u}{\partial n}\right|_{CD}=\psi_4(x),\ -1\le x\le 0 \qquad (8)$$

And 4)following gluing conditions:

$$u(x,+0)=u(x,-0)=T(x),\ -1\le x\le 1, \qquad (9)$$

$$u_y(x,+0)=u_y(x,-0)=N(x),\ -1\le x\le 1, \qquad (10)$$

$$u_{yy}(x,+0)=u_{yy}(x,-0)=M(x),\ -1\le x\le 1, \qquad (11)$$

$$u(+0,y)=u(-0,y)=\tau_3(y),\ 0\le y\le 1, \qquad (12)$$

$$u_x(+0,y)=u_x(-0,y)=\nu_3(y),\ 0\le y\le 1, \qquad (13)$$

$$u_{xx}(+0,y)=u_{xx}(-0,y)=\mu_3(y),\ 0<y<1, \qquad (14)$$

where

$$T(x)=\begin{cases}\tau_1(x), & \text{если}\ \ 0\le x\le 1,\\ \tau_2(x), & \text{если}\ \ -1\le x\le 0;\end{cases}$$

$$N(x)=\begin{cases}\nu_1(x), & \text{если}\ \ 0\le x\le 1,\\ \nu_2(x), & \text{если}\ \ -1\le x\le 0;\end{cases}$$

$$M(x)=\begin{cases}\mu_1(x), & \text{если}\ \ 0<x<1,\\ \mu_2(x), & \text{если}\ \ -1<x<0,\end{cases}$$

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$\varphi_j\left(j=\overline{1,3}\right)\psi_i\left(i=\overline{1,4}\right),$ given sufficiently smooth functions, $\tau_i, \nu_i, \mu_i \ (i=1,2,3)$ - unknown yet sufficiently smooth functions, $n-$ intrinsic normal to a straight line $x+y=0$ or $x-y=1$ and the point F has coordinates $F(-1/2,-1/2)$

II. Research task

Theorem. IF $\psi_1\in C^3[0,1]$, $\psi_2\in C^3[-1,-1/2]$, $\psi_3\in C^2[0,1]$, $\psi_4\in C^2[-1,0]$, $\varphi_1\in C^3[0,1]$, $\varphi_2\in C^3[0,1]$, $\varphi_3\in C^2[0,1]$, and the matching conditions are satisfied

$\varphi_1(0)=\psi_1(1)$, $\varphi_2(0)=\psi_2(-1)$, $\psi_3'(0)=-\psi_4'(0)$, then problem-1 admits a unique solution.

Evidence. We prove the theorem by the method of constructing a solution. For this, we rewrite equation (1) in the form

$$u_{1xx}-u_{1y}=\omega_1(bx-ay)e^{-\frac{c}{b}y},\ (x,y)\in D_1, \tag{15}$$

$$u_{ixx}-u_{iyy}=\omega_i(bx-ay)e^{-\frac{c}{b}y},\ (x,y)\in D_i\quad (i=2,3), \tag{16}$$

where the notation is introduced

$u(x,y)=u_i(x,y),\ (x,y)\in D_i\ \left(i=\overline{1,3}\right)$, moreover $\omega_i(bx-ay)\left(i=\overline{1,3}\right)-$ unknown so far sufficiently smooth functions to be determined.Let's look at the area first. We write down the solution to equation (16) $(i=2)$, satisfying conditions (9) and (10):

$$u_2(x,y)=\frac{1}{2}\left[T(x+y)+T(x-y)\right]+\frac{1}{2}\int_{x-y}^{x+y}N(t)\,dt-\frac{1}{2}\int_0^y e^{-\frac{c}{b}\eta}d\eta\int_{x-y+\eta}^{x+y-\eta}\omega_2(b\xi-a\eta)\,d\xi\ . \tag{17}$$

Substituting (17) into conditions (7) and (8), after some transformations, we find

$$\omega_2(bx-ay)=-\sqrt{2}\psi_3'\left(\frac{bx-ay-a}{b-a}\right)e^{\frac{c(bx-ay-b)}{b(b-a)}},\ a\le bx-ay\le b,$$

$$\omega_2(bx-ay)=\sqrt{2}\psi_4'\left(\frac{bx-ay-a}{b+a}\right)e^{-\frac{c(bx-ay+b)}{b(b+a)}},\ -b\le bx-ay\le a.$$

These equalities imply $\psi_3'(0)=-\psi_4'(0)$.

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Substituting (17) into (5), after some calculations, we have

$$T'(x)+N(x)=\alpha_1(x),\ -1\le x\le 1, \qquad (18)$$

where

$$\alpha_1(x)=\psi_1'\left(\frac{x+1}{2}\right)+\int_0^{\frac{x-1}{2}} e^{-\frac{c}{b}\eta}\omega_2\left(bx-(b+a)\eta\right)d\eta. \quad \tau_2'(x)+\nu_2(x)=\alpha_1(x),\ -1\le x\le 0. \qquad (19)$$

Substituting (17) into (6), after some calculations, we obtain

$$\tau_2'(x)-\nu_2(x)=\delta_1(x),\ -1\le x\le 0, \qquad (20)$$

where

$$\delta_1(x)=\psi_2'\left(\frac{x-1}{2}\right)+\int_0^{-\frac{x+1}{2}} e^{-\frac{c}{b}\eta}\omega_2\left(bx+(b-a)\eta\right)d\eta.$$

From (19) and (20) we find

$$\tau_2'(x)=\frac{1}{2}\left[\alpha_1(x)+\delta_1(x)\right], \quad \nu_2(x)=\frac{1}{2}\left[\alpha_1(x)-\delta_1(x)\right]. \qquad (21)$$

Integrating the first of equalities (21) from -1 to x, we find

$$\tau_2(x)=\frac{1}{2}\int_{-1}^{x}\left[\alpha_1(t)+\delta_1(t)\right]dt+\psi_2(-1).$$

And for $0\le x\le 1$ equation (18) takes the form

$$\tau_1'(x)+\nu_1(x)=\alpha_1(x),\ 0\le x\le 1. \qquad (22)$$

Equation (1) can be rewritten as

$$au_{1xxx}+bu_{1xxy}+cu_{1xx}-au_{1xy}-bu_{1yy}-cu_{1y}=0.$$

Passing in the last equation and in the equation

(16) $(i=2)$ to the limit at

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$y \to 0$, we have relations between unknown functions $\tau_1(x)$, $\nu_1(x)$ и $\mu_1(x)$:

$$a\tau_1'''(x)+b\nu_1''(x)+c\tau_1''(x)-a\nu_1'(x)-b\mu_1(x)-c\nu_1(x)=0,\ 0\le x\le 1, \qquad (23)$$

$$\tau_1''(x)-\mu_1(x)=\omega_2(bx),\ \ 0\le x\le 1. \qquad (24)$$

Eliminating from (22), (23), and (24) the functions

$\nu_1(x)$, $\mu_1(x)$ and integrating the resulting equation from 0 to x, we arrive at the equation

$$\tau_1''(x)+\left(1-\frac{c}{b-a}\right)\tau_1'(x)-\frac{c}{b-a}\tau_1(x)=\alpha_2(x)+k_1,\ \ 0\le x\le 1, \qquad (25)$$

where

$$\alpha_2(x)=\frac{1}{b-a}\left\{b\alpha_1'(x)-a\alpha_1(x)+\int_0^x\left[b\omega_2(bt)-c\alpha_1(t)\right]dt\right\},$$

while $k_1 -$ the unknown is constant.

When solving equation (25), there can be three cases:

1°. $c\ne-(b-a)$, $c\ne 0$;

2°. $c=-(b-a)$;

3°. $c=0$.

In case 1 °, the characteristic equation of equation (25) has two different real roots:

$\lambda_1=-1$, $\lambda_2=\dfrac{c}{b-a}$.

In the case of 2 °, the characteristic equation of equation (25) has one double real root: $\lambda_{1,2}=-1$

.

In the case of 3 °, the characteristic equation of equation (25) has two different real roots:

$\lambda_1=-1$, $\lambda_2=0$.

Consider case 1 °. Solving equation (25) under the conditions

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$$\tau_1(0)=\frac{1}{2}\int_{-1}^{0}\left[\alpha_1(t)+\delta_1(t)\right]dt+\psi_2(-1),$$

$$\tau_1'(0)=\frac{1}{2}\left[\alpha_1(0)+\delta_1(0)\right],\quad \tau_1(1)=\varphi_1(0),\qquad (26)$$

find

$$\tau_1(x)=\frac{b-a}{c+b-a}\int_0^x\left[e^{\frac{c(x-t)}{b-a}}-e^{t-x}\right]\alpha_2(t)\,dt+$$
$$+\frac{b-a}{c+b-a}k_1\left[\frac{b-a}{c}\left(e^{\frac{cx}{b-a}}-1\right)-\left(1-e^{-x}\right)\right]+k_2e^{-x}+k_3e^{\frac{cx}{b-a}},$$

where

$$k_3=\frac{b-a}{2(b-a+c)}\left\{\int_{-1}^{0}\left[\alpha_1(t)+\delta_1(t)\right]dt+2\psi_2(-1)+\alpha_1(0)+\delta_1(0)\right\},$$

$$k_2=\frac{c}{2(b-a+c)}\left\{\int_{-1}^{0}\left[\alpha_1(t)+\delta_1(t)\right]dt+2\psi_2(-1)-\alpha_1(0)-\delta_1(0)\right\},$$

$$k_1=\left[\frac{b-a}{c}\left(e^{\frac{c}{b-a}}-1\right)-\left(1-e^{-1}\right)\right]^{-1}\left\{\frac{b-a+c}{b-a}\left[\varphi_1(0)-k_2e^{-1}-k_3e^{\frac{c}{b-a}}\right]-\right.$$
$$\left.-\int_0^1\left[e^{\frac{c(1-t)}{b-a}}-e^{t-1}\right]\alpha_2(t)\,dt\right\}.$$

Consider case 2 °. Solving equation (25) under conditions (26), we have

$$\tau_1(x)=\int_0^x(x-t)e^{t-x}\alpha_2(t)\,dt+k_1\left[1-(1+x)e^{-x}\right]+(k_2+k_3x)e^{-x},$$

where

$$k_2=\frac{1}{2}\int_{-1}^{0}\left[\alpha_1(t)+\delta_1(t)\right]dt+\psi_2(-1),\quad k_3=k_2+\frac{1}{2}\left[\alpha_1(0)+\delta_1(0)\right],$$

$$k_1=\frac{1}{e-2}\left[\varphi_1(0)e-k_2-k_3-\int_0^1(1-t)e^{t-1}\alpha_2(t)\,dt\right].$$

Finally, consider case 3 °. In this case, Eq. (25) has the form

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$$\tau_1''(x)+\tau_1'(x)=\alpha_2(x)+k_1,\ 0\le x\le 1.$$

Integrating this equation from 0 to x, we arrive at the equation

$$\tau_1'(x)+\tau_1(x)=\alpha_3(x)+k_1x+k_2,\ 0\le x\le 1,$$

where

$$\alpha_3(x)=\int_0^x\alpha_2(t)\,dt.$$

Solving the last equation under conditions (26), we have

$$\tau_1(x)=\int_0^x e^{t-x}\alpha_3(t)\,dt+k_1\left(x-1-e^{-x}\right)+k_2\left(1-e^{-x}\right)+k_3e^{-x},$$

where

$$k_3=\frac{1}{2}\int_{-1}^{0}\left[\alpha_1(t)+\delta_1(t)\right]dt+\psi_2(-1),$$

$$k_2=k_3+\frac{1}{2}\left[\alpha_1(0)+\delta_1(0)\right],$$

$$k_1=\varphi_1(0)e-k_2(e-1)-k_3-\int_0^1 e^t\alpha_3(t)\,dt.$$

Now we go to the area G_3. Passing in equations (16) $(i=3)$ and (16) $(i=2)$ to the limit at $y\to 0$, get

$$\omega_{31}(bx)=\omega_2(bx),\ -1\le x\le 0.$$

Changing the argument bx to $bx-ay$, we have

$$\omega_{31}(bx-ay)=\omega_2(bx-ay),\ -b\le bx-ay\le 0.$$

Here it is set
$$\omega_3(bx-ay)=\begin{cases}\omega_{31}(bx-ay), & -b\le bx-ay\le 0,\\ \omega_{32}(bx-ay), & -b-a\le bx-ay\le -b.\end{cases}$$

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Consider the following problem:

$$\begin{cases} u_{3xx} - u_{3yy} = \omega_3 (bx - ay) e^{-\frac{c}{b}y}, \\ u_3(x,0) = \tau_2(x),\ u_{3y}(x,0) = \nu_2(x),\ \ -1 \le x \le 0, \\ u_3(-1,y) = \varphi_2(y),\ u_{3x}(-1,y) = \varphi_3(y),\ u_3(0,y) = \tau_3(y),\ 0 \le y \le 1. \end{cases}$$

We will seek a solution to this problem in the form

$$u_3(x,y) = u_{31}(x,y) + u_{32}(x,y) + u_{33}(x,y), \qquad (27)$$

where $u_{31}(x,y) -$ the solution to the problem

$$\begin{cases} u_{31xx} - u_{31yy} = 0, \\ u_{31}(x,0) = \tau_2(x),\ u_{31y}(x,0) = 0,\ -1 \le x \le 0, \\ u_{31}(-1,y) = \varphi_2(y),\ u_{31}(0,y) = \tau_3(y),\ 0 \le y \le 1; \end{cases} \qquad (28)$$

$u_{32}(x,y) -$ the solution to the problem

$$\begin{cases} u_{32xx} - u_{32yy} = 0, \\ u_{32}(x,0) = 0,\ u_{32y}(x,0) = \nu_2(x),\ -1 \le x \le 0, \\ u_{32}(-1,y) = 0,\ u_{32}(0,y) = 0,\ 0 \le y \le 1; \end{cases} \qquad (29)$$

$u_{33}(x,y) -$ the solution to the problem

$$\begin{cases} u_{33xx} - u_{33yy} = \omega_3 (bx - ay) e^{-\frac{c}{b}y}, \\ u_{33}(x,0) = 0,\ u_{33y}(x,0) = 0,\ \ -1 \le x \le 0, \\ u_{33}(-1,y) = 0,\ u_{33}(0,y) = 0,\ 0 \le y \le 1. \end{cases} \qquad (30)$$

Using the continuation method, we find solutions to problems (28) - (30). They look like

$$u_{31}(x,y) = \frac{1}{2}\left[T_2(x+y) + T_2(x-y)\right], \qquad (31)$$

Where

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$$T_2(x)=\begin{cases}2\varphi_2(-1-x)-\tau_2(-2-x),\ -2\le x\le -1,\\ \tau_2(x),\ -1\le x\le 0,\\ 2\tau_3(x)-\tau_2(-x),\ 0\le x\le 1;\end{cases}$$

$$u_{32}(x,y)=\frac{1}{2}\int_{x-y}^{x+y}N_2(t)\,dt \quad , \qquad (32)$$

Where

$$N_2(x)=\begin{cases}-\nu_2(-2-x),\ -2\le x\le -1,\\ \nu_2(x),\ -1\le x\le 0,\\ -\nu_2(-x),\ 0\le x\le 1;\end{cases}$$

It has the form $u_{33}(x,y)$

$$u_{33}(x,y)=-\frac{1}{2}\int_0^y e^{-\frac{c}{b}\eta}\,d\eta\int_{x-y+\eta}^{x+y-\eta}\Omega_3(b\xi-a\eta)\,d\xi \quad . \qquad (33)$$

The first two conditions of problem (30) are satisfied automatically. Satisfying the third of the conditions of problem (30), after simplification, we obtain

$$\int_0^y e^{-\frac{c}{b}\eta}\Omega_3\big(b(-1-y)+(b-a)\eta\big)\,d\eta=-\int_0^y e^{-\frac{c}{b}\eta}\Omega_3\big(b(y-1)-(b+a)\eta\big)\,d\eta \quad . \qquad (34)$$

Changing variables in integrals (34) and differentiating the resulting equation, after some transformations, we obtain

$$\frac{b}{b-a}\Omega_3\big(b(-1-y)\big)-\frac{2a^2}{b^2-a^2}\Omega_3(-b-ay)-\frac{2a^2c}{(b+a)^2(b-a)}\int_0^y\Omega_3(-b-at)\,e^{-\frac{c(by+at)}{b(b+a)}}\,dt=$$

$$=-\frac{b}{b+a}\omega_{31}\big(b(y-1)\big)+\frac{2abc}{(b+a)^2(b-a)}\int_{-1}^{y-1}\omega_{31}(bz)\,e^{-\frac{c(y-1-z)}{b+a}}\,dz \quad . \qquad (35)$$

Setting in (33) $x\to 0$, after some transformations, we have

$$\int_0^y e^{-\frac{c}{b}\eta}\Omega_3\big(by-(b+a)\eta\big)\,d\eta=-\int_0^y e^{-\frac{c}{b}\eta}\Omega_3\big((b-a)\eta-by\big)\,d\eta \quad . \qquad (36)$$

Changing variables in the integrals of equality (36) and differentiating the resulting equation, after long transformations, we find

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$$\Omega_3(by)=\frac{2a^2}{b(b-a)}\omega_{31}(-ay)e^{-\frac{c}{b}\eta}-\frac{b+a}{b-a}\omega_{31}(-by)-\frac{2ac}{(b-a)^2}\int_{\frac{a}{b}y}^{y}e^{-\frac{c(y-z)}{b-a}}\omega_{31}(-bz)dz \quad . \quad (37)$$

Substituting (31), (32), and (33) into (27), we have

$$u_3(x,y)=\frac{1}{2}\left[T_2(x+y)+T_2(x-y)\right]+\frac{1}{2}\int_{x-y}^{x+y}N_2(t)dt-\frac{1}{2}\int_0^y e^{-\frac{c}{b}\eta}d\eta\int_{x-y+\eta}^{x+y-\eta}\Omega_3(b\xi-a\eta)d\xi \quad . \quad (38)$$

Differentiating this solution with respect to x, we have

$$u_{3x}(x,y)=\frac{1}{2}\left[T_2'(x+y)+T_2'(x-y)\right]+\frac{1}{2}\left[N_2(x+y)-N_2(x-y)\right]-$$

$$-\frac{1}{2}\int_0^y e^{-\frac{c}{b}\eta}\Omega_3\left(b(x+y)-(b+a)\eta\right)d\eta+\frac{1}{2}\int_0^y e^{-\frac{c}{b}\eta}\Omega_3\left(b(x-y)+(b-a)\eta\right)d\eta \quad . \quad (39)$$

Letting x tend to unity in (39), after long calculations and transformations, we find

$$\Omega_3(-b-ay)=\left\{\frac{b+a}{a}\left[\tau_2''(y-1)+\nu_2'(y-1)-\varphi_2''(y)-\varphi_3'(y)\right]-\frac{b}{a}\omega_{31}\left(b(y-1)\right)-\right.$$

$$\left.-\frac{c}{a}\left[\tau_2'(y-1)+\nu_2(y-1)-\varphi_2'(y)-\varphi_3(y)\right]\right\}e^{\frac{c}{b}y} .$$

Substituting the last equality in (35), we find

$$\Omega_3\left(b(-1-y)\right)=\left\{\frac{2a}{b}\left[\tau_2''(y-1)+\nu_2'(y-1)-\varphi_2''(y)-\varphi_3'(y)\right]-\frac{2a}{b+a}\omega_{31}\left(b(y-1)\right)-\right.$$

$$\left.-\frac{2ac}{b(b+a)}\left[\tau_2'(y-1)+\nu_2(y-1)-\varphi_2'(y)-\varphi_3(y)\right]\right\}e^{\frac{c}{b}y}-\frac{b-a}{b+a}\omega_{31}\left(b(y-1)\right)+$$

$$+\frac{2c}{b+a}\int_0^y\left[\tau_2''(t-1)+\nu_2'(t-1)-\varphi_2''(t)-\varphi_3'(t)\right]e^{\frac{c(t-y)}{b+a}}dt-\frac{2bc}{(b+a)^2}\int_0^y\omega_{31}\left(b(t-1)\right)e^{\frac{c(t-y)}{b+a}}dt-$$

$$-\frac{2c^2}{(b+a)^2}\int_0^y\left[\tau_2'(t-1)+\nu_2(t-1)-\varphi_2'(t)-\varphi_3(t)\right]e^{\frac{c(t-y)}{b+a}}dt+\frac{2ac}{(b+a)^2}\int_{-1}^{y-1}\omega_{31}(bz)e^{-\frac{c(y-1-z)}{b+a}}dz .$$

Now, letting x tend to zero in (33), taking into account (36), after long calculations and transformations, we obtain the relation

$$\nu_3(y)=\tau_3'(y)+\beta_1(y) , \quad (40)$$

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Where

$$\beta_1(y)=\tau_2'(-y)-\nu_2(y)+\frac{b}{b-a}\int_{\frac{a}{b}y}^{y} e^{-\frac{c(y-z)}{b-a}}\omega_{31}(-bz)dz.$$

Now go to G_1. Passing in equation (15) to the limit at $y\to 0$, we find

$$\omega_{12}(bx)=\tau_1''(x)-\nu_1(x),\ 0\le x\le 1, \tag{41}$$

where it should be

$$\omega_1(bx-ay)=\begin{cases}\omega_{11}(bx-ay), & -a\le bx-ay\le 0,\\ \omega_{12}(bx-ay), & 0\le bx-ay\le b.\end{cases}$$

And passing in equations (15) and (16) $(i=3)$ to the limit at $x\to 0$, we obtain the relations

$$\mu_3(y)-\tau_3'(y)=\omega_{11}(-ay)e^{-\frac{c}{b}y},\qquad \mu_3(y)-\tau_3''(y)=\omega_{31}(-ay)e^{-\frac{c}{b}y}.$$

Eliminating the function from these relations, we obtain

$$\omega_{11}(-ay)=\left[\tau_3''(y)-\tau_3'(y)\right]e^{\frac{c}{b}y}+\omega_{31}(-ay).$$

Next, we write down the solution of equation (15) satisfying conditions (2), (9), (12):

$$u_1(x,y)=\int_0^y\tau_2(\eta)G_\xi(x,y;0,\eta)d\eta-\int_0^y\tau_3(\eta)G_\xi(x,y;1,\eta)d\eta+\int_0^1\tau_1(\xi)G(x,y;\xi,0)d\xi-$$

$$-\int_0^y e^{-\frac{c}{b}\eta}d\eta\int_0^{\frac{a}{b}\eta}\omega_{11}(b\xi-a\eta)G(x,y;\xi,\eta)d\xi-\int_0^y e^{-\frac{c}{b}\eta}d\eta\int_{\frac{a}{b}\eta}^{1}\omega_{12}(b\xi-a\eta)G(x,y;\xi,\eta)d\xi,$$

where $\left.\begin{matrix}G(x,y;\xi,\eta)\\ N(x,y;\xi,\eta)\end{matrix}\right\}=\dfrac{1}{2\sqrt{\pi(y-\eta)}}\sum\limits_{n=-\infty}^{+\infty}\left\{\exp\left[-\dfrac{(x-\xi-2n)^2}{4(y-\eta)}\right]\mp\exp\left[-\dfrac{(x+\xi-2n)^2}{4(y-\eta)}\right]\right\}$ - Green's functions of the first and second boundary value problems for the Fourier equation.

Differentiating this solution with respect to x and tending x to zero, taking into account (40), (41) and (42) after lengthy calculations and transformations, we obtain the Volterra integral equation of the second kind with respect to the unknown function $\tau_3'(y)$:

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$$\tau_3'(y)+\int_0^y K(y,\eta)\tau_3'(\eta)d\eta = g(y), \qquad (43)$$

where $K(y,\eta)$, $g(y)-$ -known functions, and the kernel $K(y,\eta)$ has a weak singularity $\left(\frac{1}{2}\right)$, and $g(y)-$ is continuous. Therefore, equation (45) admits a unique solution in the class of continuous functions. Solving this equation, we find the function $\tau_3'(y)$ and thus the functions $\tau_3(y)$, $\nu_3(y)$, $\omega_{11}(bx-ay)$, $u_1(x,y)$, $u_3(x,y)$.

Thus, we have determined the solution to Problem 1 completely.

Comment. Similar problems for equations of the third and fourth orders were studied in [14] - [18].

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