

## CALCULATION OF THE EXACT INTEGRAL BY THE MONTE CARLO METHOD

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**Abstract:** The most important aspect of constructing Monte Carlo methods is to reduce the problem to the calculation of mathematical expectations. Since mathematical expectations are often simple integrals, the central place in the theory of the Monte Carlo method is occupied by methods of calculating integrals.

**Key words :** Definite integral, random variable, mathematical expectation, Chebyshev's theorem, statistics of experiments, surface bounded by lines, unit square, uniformly distributed, curved trapezoidal surface, random points.

1. In solving many problems in nature, it is difficult or impossible to obtain an exact solution. The Monte Carlo method is a numerical method for studying many problems and obtaining results. The essence of this method is as follows: the process is described by a mathematical model using a generator of random variables, the model is calculated many times, and based on the obtained data, the probability characteristics of the considered process are calculated.

The Monte Carlo method is used to solve problems related to various fields of physics, chemistry, mathematics, economics, optimization, control theory, etc.

$\int_0^1 \varphi(t) dt$  we use the Monte Carlo method to calculate the exact integral. Here  $t$  is a uniformly distributed random variable whose density function is  $P(t)$ :

$$P(t) = \begin{cases} 0, & t < 0 \\ 1, & 0 < t \leq 1 \\ 0, & t > 1 \end{cases}$$

let it be The mathematical expectation of a random  $\varphi(t)$  function is defined as:

$$M(\varphi(t)) = \int_0^1 \varphi(t) \cdot P(t) dt$$

$P(t)$  based  $0 \leq t \leq 1$  on the value of

$$M(\varphi(t)) = \int_0^1 \varphi(t) dt \quad (1.1)$$

We calculate the approximate value of the mathematical expectation. Let  $N$  experiments have  $tN$  random variables  $t_1, t_2, \dots, t_N$  values. These values can be obtained from the table of random numbers [10]. In this case, the  $M(\varphi(t))$  mathematical expectation value is found from the following equation based on Chebyshev's theorem.

$$M(\varphi(t)) \approx \frac{1}{N} \sum_{i=1}^N \varphi(t_i) \quad (1.2)$$

(1.2) and (1.2) on the basis of equations

$$\int_0^1 \varphi(t)dt \approx \frac{1}{N} \sum_{i=1}^N \varphi(t_i) \tag{1.3}$$

2 . Let's see the general case.  $\int_a^b f(x)dx$  be required to calculate an integer.  $x = a +$

$(b-a)t$  with equality  $t$  we pass to the variable. In this case

$$\int_a^b f(x)dx = (b-a) \int_0^1 \varphi(t)dt \tag{1.4}$$

here  $\varphi(t) = f(a + (b-a)t)$ . Based on the formula (1.3), we calculate the right side of the formula (1.4).

$$\int_a^b f(x)dx \approx \frac{b-a}{N} \sum_{i=1}^N \varphi(t_i) \text{ or } \int_a^b f(x)dx \approx \frac{b-a}{N} \sum_{i=1}^N f(x_i) \tag{1.5}$$

here

$$x_i = a + (b-a)t_i, (i = 1, 2, \dots, n).$$

We make a table for calculating the integral.

Table 1

$i$	$t_i$	$x_i = a + (b-a)t_i$	$f(x_i)$
1	$T_1$	$x_1$	$f(x_1)$
2	$t_2$	$x_2$	$f(x_2)$
...	...	...	...
$N$	$t_N$	$x_N$	$f(x_N)$

In this method, calculation of the exact integral based on the formula (1.5) by the **Monte Carlo method** is calculated in simple methods of statistics of experiments.

**We will see how to calculate the exact integral based on the Monte Carlo method:**

$$I = \int_a^b f(x)dx$$

The geometric meaning of the definite integral: is  $x = a, x = b, y = 0, y = f(x)$  equal to the surface bounded by the lines, if  $f(x)$  the function  $[a, b]$  is continuous and positive.

Now  $x = a, x = b, y = 0, y = M$  ( $M \geq \max f(x), [a, b]$ ) let's see the quadrilateral, (Fig. 1). If the  $f(x) \geq 0$  inequality  $[a, b]$  does not hold at all points of , we use the following expression:

$$\int_a^b f(x)dx = \int_a^b [f(x) + h]dx - h(b-a)$$

's choose  $f(x) + h \geq 0$  for  $h > 0$  that  $x \in [a, b]$ .

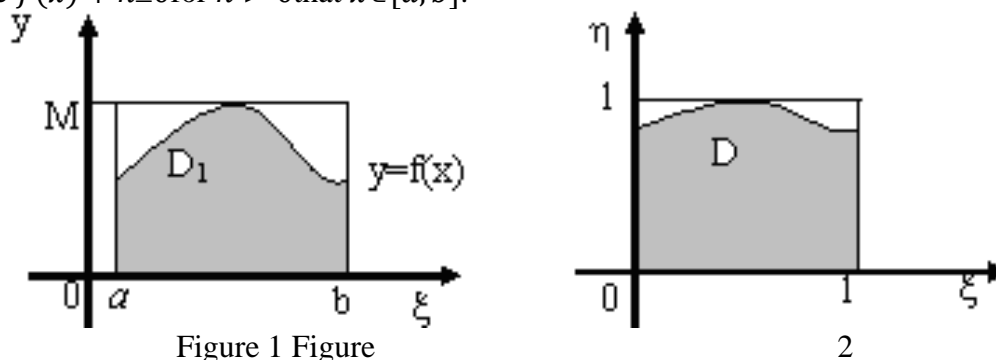


Figure 1 Figure

This method is [0,1]based on a table of random numbers, which also belongs to the cross section. Therefore, it is necessary to switch from variables  $x, y$  to variables such  $\xi$  that the unit  $\eta$  area  $0 \leq \xi \leq 1, 0 \leq \eta \leq 1$  is replaced by area  $D$ , which is a square (Fig. 2). for this

$$x = a + (b - a) \xi, y = M \eta$$

we will exchange. Where  $dx = (b - a)d \xi$  and  $x \in (a, b)$  is  $\xi \in (0, 1)$ . The given integral is:

$$I = (b - a)M \int_a^b \varphi(\xi)d\xi \tag{1.6}$$

in this

$$\varphi(\xi) = \frac{1}{M} f[a + (b - a)\xi] \tag{1.7}$$

From equation (1.7)  $f(x) = M \varphi(\xi)$ .

$(\xi_1, \eta_1), (\xi_2, \eta_2), \dots, (\xi_N, \eta_N)$  with unit square uniform distribution  $\xi$ . Suppose that  $n$  points fall into the sphere  $D$ . Since the random points are uniformly distributed

$$\frac{n}{N} \xrightarrow{\text{ehtimollik}} \frac{\int_0^1 \varphi(\xi)d\xi}{1}$$

where the value of 1 unit square area. In this case

$$\int_0^1 \varphi(\xi)d\xi \approx \frac{n}{N} \tag{1.8}$$

Based on equations (1.7) and (1.8):

$$\int_0^1 f(x)dx \approx \frac{(b - a)nM}{N} \tag{1.9}$$

**This is the formula for calculating the exact integral using the Monte Carlo method.**

From the formula (1.9) we can write the following:

$$\frac{\int_0^1 f(x)dx}{M(b - a)} \approx \frac{n}{N}$$

the curved trapezoid  $D_1$  to the surface of the rectangle (Fig. 1) is approximately equal to the ratio of the number of random points falling on the area  $D_1$  to the number of random points falling on the four corners.

We write the table of approximate calculation of the definite integral with the formula (1.9).

Table 2

$I$	$\xi_i$	$\eta_i$	$x_i = a + (b - a) \xi_i$	$y_i = M \eta_i$	$U_i = f(x_i)$
1	$\xi_1$	$\eta_1$	$X_1$	$y_1$	$f(x_1)$
2	$\xi_2$	$\eta_2$	$x_2$	$y_2$	$f(x_2)$
·	·	·	·	·	·
·	·	·	·	·	·
n	$\xi_N$	$\eta_N$	$x_N$	$y_N$	$f(x_N)$

From  $Y_i (i = 1, 2, \dots, N)$  it is necessary to choose those that satisfy the condition  $y_i < Y_i$ . The number of these will be  $n$ .

1 :  $I = \int_2^3 (x^2 + x^3)dx$  we calculate the integral by formula (1.9).

Here  $a = 2$ ,  $b = 3$ ,  $\max_{2 \leq x \leq 3} (x^2 + x^3) = 36$  will be From this  $2 \leq x \leq 3$   $x=2+\xi$ ,  $y=36\eta$ . From the table of random numbers ( $\xi, \eta$ ) we get 20 ( $N=20$ ). The calculation table will be as follows.

I	$\xi_i$	$\eta_i$	$x_{i=2+\xi_i}$	$y_{i=36\eta_i}$	$x_i^2$	$x_i^3$	$Y_i = x_i^2 + x_i^3$
1	0.857	0.457	2,857	<u>16,452</u>	8,162	23,319	31,481
2	0.499	0.762	2,499	27,432	6,245	15,606	21,851
3	0.431	0.608	2,431	25,128	5,910	14,367	20,727
4	0.038	0.558	2,038	20,088	4,153	8,464	12,617
5	0.651	0.573	2,653	<u>20,628</u>	7,038	18,672	25,710
6	0.609	0.179	2,609	<u>6,444</u>	6,807	17,759	24,566
7	0.974	0.011	2,974	<u>0,396</u>	8,845	26,305	35,150
8	0.098	0.805	2,098	28,980	4,402	9,235	13,637
9	0.316	0.296	2,516	<u>10,656</u>	6,330	15,926	22,256
10	0.149	0.815	2,140	29,340	4,618	9,924	14,542
11	0.070	0.692	2,070	24,912	4,285	8,870	13,155
12	0.696	0.203	2,696	<u>7,308</u>	7,266	15,595	26,863
13	0.350	0.900	2,350	32,400	5,523	12,979	18,502
14	0.451	0.318	0,451	<u>11,448</u>	6,007	14,723	20,730
15	0.798	0.111	2,798	<u>3,906</u>	7,829	21,906	20,736
16	0.933	0.199	2,933	<u>7,164</u>	8,602	25,230	33,832
17	0.183	0.421	2,183	<u>15,155</u>	4,765	10,402	15,167
18	0.338	0.104	2,338	<u>3,744</u>	5,466	12,780	18,246
19	0.190	0.150	2,190	<u>5,400</u>	4,706	10,503	15,299
20	0.449	0.320	2,449	<u>11,520</u>	5,998	14,689	20687

We see from the table that the number of values (points) satisfying the condition  $y_i < Y_i$  is equal to  $n=13$ . (1.9) according to the formula:

$$I \approx \frac{36 \cdot 13}{20} = 23,4$$

One of the first ways to use random numbers was to calculate the integral. Let us generate uniformly distributed random numbers  $x_1, \dots, x_n$  in the interval  $a$  and  $b$ , then the approximation of the solution is found as follows:

$$\int_a^b f(x) dx = \frac{b-a}{n} \sum_{i=1}^n f(x_i) \tag{1.10}$$

This method is usually called Monte Carlo integration. We can express the expression (1.10) as a subroutine:

```
import random as random_number

def MCint (f, a, b, n):
s = 0
```

```

    for i in range (n):
x = random_number . uniform(a, b)
s += f(x)
I = ( float (b - a) / n) * s
    return I

```

Usually, n is given with a large number so that the method achieves sufficient accuracy, so the vectorized version is more convenient:

```

from numpy import *

def MCint_vec ( f , a _ b , n ):
    x = random . uniform ( a , b , n )
    s = sum ( f ( x ))
    I = ( float ( b - a ) / n ) * s
    return I

```

Let's consider Monte Carlo integration on the example of a simple linear function  $f(x)=2x+3$ , we take the limits of integration from 1 to 2. It is more interesting to see how the method can solve the problem for different n. We see the evaluation in the following MCint method, slightly modified:

```

def MCint2 ( f , a _ b , n ):
    s = 0

    I = zeros ( n )
    for k in range ( 1 , n + 1 ):
        x = random_number . uniform ( a , b )
        s += f ( x )
        I [ k - 1 ] = ( float ( b - a ) / k ) * s
    return I

```

It appears that k' varies from 1 to n. Given that n can be very large, the array I can occupy or fill memory. Therefore, only every Nth approximation value should be recorded. This can be solved using the familiar residue detection function:

```

for k in range ( 1 , n + 1 ):
    ...
    if k % N == 0 :
        # store

```

Thus, every time k is divided by N without a remainder, we write the value (in our case from each hundred). The corresponding function is shown below.

```

def MCint3 ( f , a _ b , n _ N = 100 ):
    '''Stores every Nth approximation of the integral in array I
and k
writes the corresponding value of
    s = 0

```

```

I_values = []
k_values = []
for k in range ( 1 , n + 1 ):
    x = random_number . uniform ( a , b )
    s += f ( x )
    if k % N == 0 :
        I = ( float ( b - a ) / k ) * s
        I_values . append ( I )
        k_values . append ( k )
return k_values , I_values

```

We now have a tool to see how the Monte Carlo integration error changes as n increases. The finished program looks like this, the output of the program (randomness may vary slightly) is shown below:

```

import random as random_number
import matplotlib.pyplot as plt
from numpy import array

def MCint3 ( f , a _ b , n _ N = 100 ):
    '''Stores every Nth approximation of the integral in array I
and k
writes the corresponding value of
    s = 0

    I_values = []
    k_values = []
    for k in range ( 1 , n + 1 ):
        x = random_number . uniform ( a , b )
        s += f ( x )
        if k % N == 0 :
            I = ( float ( b - a ) / k ) * s
            I_values . append ( I )
            k_values . append ( k )
    return k_values , I_values

def f1 ( x ):
    return 2 + 3 * x

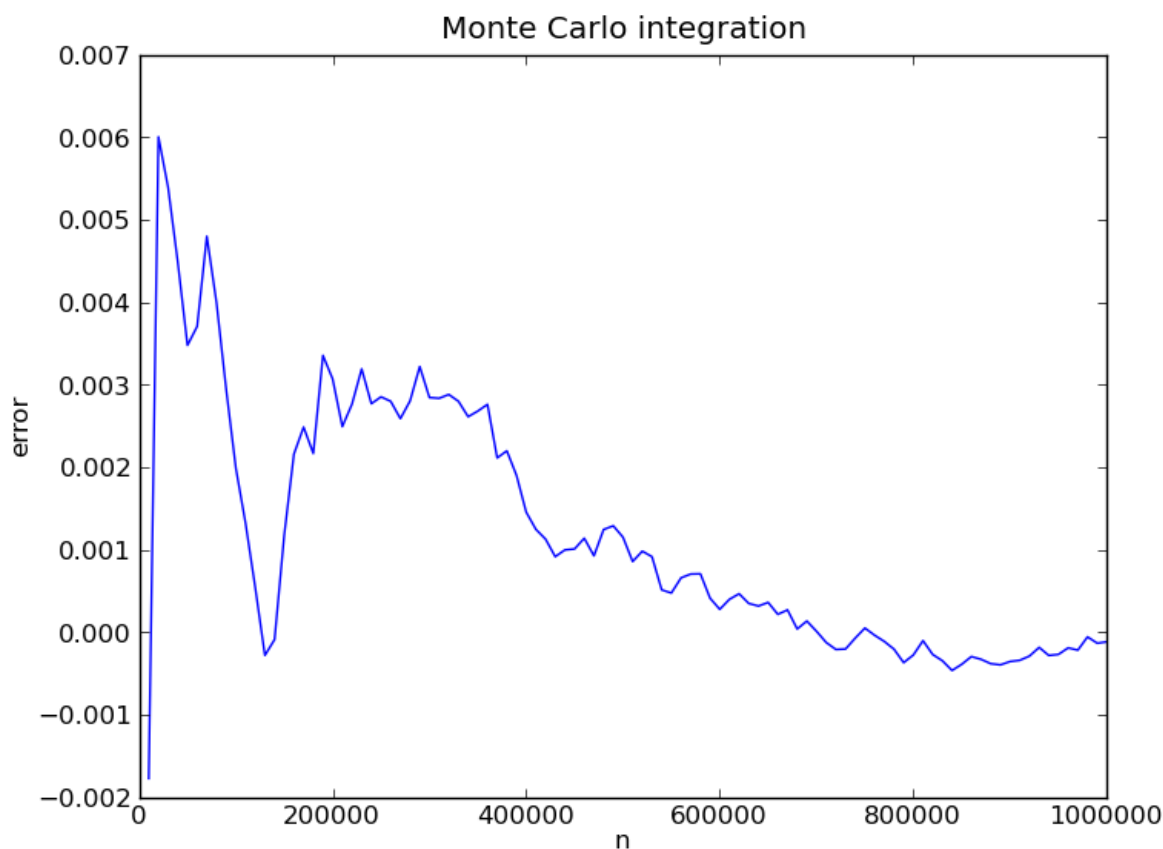
k , I = MCint3 ( f1 , 1 , 2 , 1000000 , N = 10000 )
error = 6.5 - array ( I )

plt . title ( 'Monte Carlo integration' )
plt . xlabel ( 'n' )
plt . ylabel ( 'error' )
plt . plot ( k , error )

```

```
plt . show ()
```

The result:



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