Teaching students in math circles to solve certain equations by bringing them into a system of equations.

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Annotation: This article describes how to solve some equations in a system of equations of equal strength, and gives examples of their solutions.
Keywords: trigonometry, equation, artificial, substitution, method.

Mathematics circles, mathematics is the main type of extracurricular work. Circles are interested in science in students and serve to improve their mathematical thinking , skills, quality of mathematical training below we will learn to solve some equations by quoting them into a system of equations. This further increases the circle of thinking of students.

1. $f_1^2(x)+f_2^2(x)+...+f_k^2(x)=0$ or $|f_1(x)|+|f_2(x)|+...+|f_k(x)|=0$ solving equations in the form of. $f_1^2(x)+f_2^2(x)+...+f_k^2(x)=0$ (1)

$|f_1^2(x)|+|f_2^2(x)|+...+|f_k(x)|=0$ (2)

equations in appearance

$$\begin{cases} f_1(x)=0 \\ \vdots \\ f_k(x)=0 \end{cases}$$ (3) strong equal to the system of equations.

1-example. $x^4+5\cdot 4^x+4x^2\cdot 2^x-2\cdot 2^x+1=0$ (4) solve the equation.

Solution. (4) we write the equation in the following form.

$$(x^2+2\cdot 2^x)^2+(2^x-1)^2=0 \qquad (5)$$

From this (5) equation $\begin{cases} x^2+2\cdot 2^x=0 \\ 2^x-1=0 \end{cases}$ (6) strong equal to the system of equations. (6) Equation 2 of the system has a single solution, which does not satisfy the first equation of the system. As a result (6) the system does not have a solution

2-example. $\sqrt{x^2-6x+9}+\sqrt{\log^2_{\frac{1}{7}}(x^2-4x+4)}=0$ (7) solve the equation.

Solution. (7) we write the equation in the following form.

$$|x-3|+\left|\log_{\frac{1}{7}}(x^2-4x+4)\right|=0$$

This equation is strong, equal to the following system of equations. $\begin{cases} x-3=0 \\ \log_{\frac{1}{7}}(x^2-4x+4)=0 \end{cases}$ (8)

This is the solution to the first of the equations . Verification indicates that this number is also a solution to the second equation. The result is the solution to the given equation (7).

A: $x=3$.

(3) let's look at a number of other equations that are brought to the system.

Example 3. $\log_2(1+\sqrt{x^4+x^2})+\log_2(1+x^2)=0$ (9) solve the equation.

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Solution. For optional $\begin{cases}\log_2(1+\sqrt{x^4+x^2})\geq 0\\ \log_2(1+x^2)\geq 0\end{cases}$

inequalities are appropriate. Therefore, equation (9) is strong equal to the following system of equations.

$$\begin{cases}\log_2(1+\sqrt{x^4+x^2})=0\\ \log_2(1+x^2)=0\end{cases}$$

system single $x=0$ has a solution.

A: $x=0$.

2. Using the delimitation of the function.

If $f(x)=g(x)$ (10) when solving equations, a M all belonging to the collection x 's for $f(x)\leq A$ and $g(x)\geq A$

if inequalities are appropriate, then M in a set (10), the equation will be strong equal to the following system of equations.

$$\begin{cases}f(x)=A\\ g(x)=A\end{cases} \qquad (11)$$

4-example. $4x^2+4x+17=\dfrac{12}{x^2-x+1}$

solve the equation.

Solution. We write this equation in the following form.

$$\left(x+\frac{1}{2}\right)^2+4=\frac{3}{\left(x-\frac{1}{2}\right)^2+\frac{3}{4}} \qquad (12)$$

as you can see, optional x for the real number

$$g(x)=\left(x+\frac{1}{2}\right)^2+4\geq 0; \qquad f(x)=\frac{3}{\left(x-\frac{1}{2}\right)^2+\frac{3}{4}}\leq 4.$$

As a result, the equation (12) is strong equal to the following system of equations.

$$\begin{cases}\left(x+\frac{1}{2}\right)^2+4=4\\ \left(x-\frac{1}{2}\right)^2+\frac{3}{4}=\frac{3}{4}\end{cases}.$$

This system does not have a solution, so the givem equation will not have a solution.

A: $\varnothing$.

5-example. $\cos^2(x\sin x)=1+\log_5^2\sqrt{x^2+x+1}$ (13) solve the equation.

Solution. (13) the equation is all true x 's defined for. Optional x for $\cos^2(x\sin x)\leq 1, \quad 1+\log_5^2\sqrt{x^2+x+1}\geq 1.$

As a result, the equation (13) is strong equal to the following system of equations.

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$$\begin{cases}\cos^2(x\sin x)=1\\ \log_5^2\sqrt{x^2+x+1}=0\end{cases} \quad (14)$$

(2) solution of Equation 2 of the system $x=0$ va $x=-1$. Of these values, Equation 1 is only $x=0$ satisfies. Say, $x=0$ since it is the only solution to a given equation.

A: 0.

6-example. $\cos^7 x+\sin^5 x=1$ (15) solve the equation.

Solution. $\cos^2 x+\sin^2 x=1$ since (15) we write the equation in the following form $\cos^7 x+\sin^5 x=\cos^2 x+\sin^2 x$ or

$$\cos^2 x(\cos^5 x-1)=\sin^2 x(1-\sin^3 x) \quad (16)$$

optional x for $\sin^2 x\geq 0$, $\cos^2 x\geq 0$, $\cos^5 x-1\leq 0$, $1-\sin^3 x\geq 0$ since the equation (16) is equal to the following system strong

$$\begin{cases}\cos^2 x(\cos^5 x-1)=0\\ \sin^2 x(1-\sin^3 x)=0\end{cases} \quad (17)$$

(17) the system is powerful, equal to the set of the following system of equations.

$$\begin{cases}\cos x=0,\\ \sin x=1,\end{cases} \quad \begin{cases}\sin x=0,\\ \cos x=1\end{cases} \quad (18)$$

Solution of the first system $x=\frac{\pi}{2}+2\pi k,\quad k\in z$, solution of the second system $x=2\pi m,\quad m\in z$. All these solutions will be the solution to the given equation.

A: $x=2\pi m,\qquad x=\frac{\pi}{2}+2\pi k,\quad m,k\in z$.

3. Using the properties of Sine and cosine functions.

Solving many trigonometric equations can be brought to solving a system of equations. An example of such equations is the following equations.

$$\sin\alpha x\cdot\sin\beta x=\pm 1$$

$$\sin\alpha x\cdot\cos\beta x=\pm 1$$

$$A(\sin\alpha x)^m+B(\cos\beta x)^n=\pm(|A|+|B|)$$

$$A(\sin\alpha x)^m+B(\sin\beta x)^n=\pm(|A|+|B|) \quad (19)$$

bunda α, β, A, B berilgan haqiqiy sonlar, n va m - berilgan natural sonlar. Bunday tenglamalarni yechishda sinusning quyidagi xossasidan foydalaniladi: agar biror x_0 soni uchun qat'iy $|\sin\alpha x_0|<1$ tengsizlik o'rinli bo'lsa,u holda x_0 soni (19) tenglamalardan birortasining ham yechimi bo'lmaydi. Xuddi shuningdek

$$\cos\alpha x\cdot\cos\beta x=\pm 1$$

$$A(\sin\alpha x)^m+B(\cos\beta x)^n=\pm(|A|+|B|)$$

tenglamalarni yechishda kosinus xossasidan foydalaniladi: agar biror x_0 soni uchun qat'iy $|\cos\alpha x_0|<1$ tengsizlik o'rinli bo'lsa, u holda x_0 soni bu tenglamalardan birortasining ham yechimi bo'lmaydi.

7-misol. $\sin x\cdot\cos 4x=1$ (20) tenglamani yeching.

Yechish. Agar x_0 (20) tenglamaning yechimi bo'lsa, u holda yo $\sin x_0=1$ yoki $\sin x_0=-1$ bo'ladi. Haqiqatan ham agar $|\sin x_0|<1$ bo'lsa (20) tenglamadan $|\cos 4x_0|>1$ bo'lishi kerak edi, ammo bu bo'lishi mumkin emas. Agar $\sin x_0=1$ bo'lsa (20) tenglamadan $\cos 4x_0=1$ ekanligi, agar $\sin x_0=-1$ bo'lsa,

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$\cos 4x_0 = -1$ ekanligi kelib chiqadi. Natijada (20) tenglamaning ixtiyoriy yechimi quyidagi 2 ta sistemalardan birining yechimi bo‘ladi.

$$\begin{cases} \sin x_0 = 1 \\ \cos 4x_0 = 1 \end{cases} \qquad (21)$$

$$\begin{cases} \sin x_0 = -1 \\ \cos 4x_0 = -1 \end{cases} \qquad (22)$$

It can be easily seen that the optional solution of systems (21) and (22) is the solution of equation (20). As a result (20) tenlama (21) and (22) are strong, equal to the complex of systems of equations. We will solve these systems.

(21) from the first equation of the system

$$x = \frac{\pi}{2} + 2\pi k; \quad k \in z .$$

All this will satisfy the second equation of this system and (21) will be the solution of the system. (22) the first equation of the system $x = \frac{3\pi}{2} + 2\pi e; \quad e \in z$ has a solution.

Any of these numbers does not satisfy the second equation of this system. Therefore (22) the system does not have a solution. Hence, the solution of the given equation (20) coincides with the solution of the system (21).

A: $x = \frac{\pi}{2} + 2\pi k; \quad k \in z .$

8-example. $3\cos^4 2x - 2\sin^5 x = 5$ (23) solve the equation.

Solution. If x_0 (23) if the solution of the equation is then $|\cos 2x_0| = 1,$ otherwise $|\sin x_0| > 1$ inequality must be appropriate, which is not possible.

However, if $|\cos 2x_0| = 1$ if (23) from the equation $\sin x_0 = -1$. Therefore, the arbitrary solution of equation (23) will be the solution of the following system.

$$\begin{cases} \sin x = -1 \\ |\cos 2x| = 1 \end{cases} \qquad (24)$$

(24) the arbitrary solution of the system (23) will be the solution of the equation. Hence, equation (23) is equal to System (24) strong. (24) the first equation of the system

$x = \frac{3\pi}{2} + 2\pi e, \quad e \in z$ had a solution.

All this satisfies Equation 2 of the system (24). As a result, it will be the solution to equation (23).

A: $x = \frac{3\pi}{2} + 2\pi e, \quad e \in z .$

9-example. $\cos^3 3x + \cos^{11} 7x = -2$ (25) solve the equation.

Solution. If x_0 If there is a solution to the equation (25), then $\cos 3x_0 = -1$ (otherwise $\cos 7x_0 < -1$ can not be). Say, $\cos 7x_0 = -1$. As a result, an optional solution to equation (25) will be the solution of the following system.

$$\begin{cases} \cos 3x = -1 \\ \cos 7x = -1 \end{cases} \qquad (26)$$

(26) the arbitrary solution of the system (25) will be the solution of the equation. Therefore, equation (25) is equal to system (26) strong. (26) Equation 1 of the system

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$x_k=\frac{\pi}{3}+\frac{2\pi k}{3}, \quad k\in z$ has a solution.

From these solutions we find those that satisfy Equation 2 of the system (26). These are those that satisfy the following equality $m\in z$ numbers.

$$\frac{7\pi}{3}+\frac{14\pi k}{3}=\pi+2\pi m \qquad (27)$$

(27) we write ni in the following view

$$k=\frac{3m-2}{7} \qquad (28)$$

k and m since S are integers (28) equality $m=7t+3, \quad t\in z$ at appropriate, but in that $k=3t+1, \quad t\in z$.

So (26) the solution of the system is x_k 's, $k=3t+1, \quad t\in z$.

$$x=\frac{\pi}{3}+2\pi t+\frac{2\pi}{3}, \qquad t\in z.$$

A: $x=\pi+2\pi t, \quad t\in z$.

4.Folation from numerical inequalities.

In some cases it is possible to substitute an equation into an equally powerful system, supporting a number of inequalities to a part of the equation. An example of such inequalities are two positives a va b between the mid-arithmetic and mid-geometric of the numbers $\frac{a+b}{2}\geq\sqrt{ab}$ we take the link, the sign of equality $a=b$ At Reasonable.

In most cases, it is convenient to use the result of the following inequalities

a>0 da $a+\frac{1}{a}\geq 2$, *a=1 da* $a+\frac{1}{a}=2$, $a<0$ *da* $a+\frac{1}{a}\leq -2$, *a=-1 da* $a+\frac{1}{a}=-2$.

10-exmple. $\sqrt{x^2+2x+4}+\frac{4}{\sqrt{x^2+2x+4}}=4-\log_3^4(x^2+x^4+1)$ (29)

Solve the equation.

Solution. The field of determination of the equation is all real numbers. (29) the left part of the equation

$$2\left(\frac{\sqrt{x^2+2x+4}}{2}+\frac{2}{\sqrt{x^2+2x+4}}\right)$$

we write in appearance.

We assume that it is not smaller than 4. The sum of 2 mutually inverse positive numbers is only $x=0$ da 4 is equal to. At one time $x=0$ at the right part of the equation is also 4. $x\neq 0$ at smaller than 4. As a result $x=0$ (29) is the only solution to the equation.

A: $x=0$.

11-example. $\left(\frac{1}{\sin^8}+\frac{1}{\cos^2 2x}\right)\left(\sin^8 x+\cos^2 2x\right)=4\cos^2\sqrt{\frac{\pi^2}{4}-x^2}$ (30) solve the equation.

Solution. Optional positive a va b for the numbers $\left(\frac{1}{a}+\frac{1}{b}\right)(a+b)\geq 4$ (31) we prove that inequality is appropriate. Before $\frac{1}{a}$ va $\frac{1}{b}$ for the numbers, then a and b support the inequality between the arithmetic

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mean and the geometric mean for the numbers $\frac{\frac{1}{a}+\frac{1}{b}}{2} \geq \sqrt{\frac{1}{a}\cdot\frac{1}{b}}$ va $\frac{a+b}{2} \geq \sqrt{ab}$ we form the. From this $\frac{1}{2}\left(\frac{1}{a}+\frac{1}{b}\right)\left(\frac{a+b}{2}\right) \geq 1$ $\left(\frac{1}{a}+\frac{1}{b}\right)(a+b) \geq 4$.

(30) in the field of determination of the equation $\sin^8 x > 0, \quad \cos^2 2x > 0$

for being

We see that the left part of the equation (31) in support of inequality (30) is not smaller than 4. At the same time in the field of determination of equation (30)

$$4\cos^2\sqrt{\frac{\pi^2}{4}-x^2} \leq 4$$

As a result, the equation (30) is strong equal to the following system of equations.

$$\begin{cases} \left(\frac{1}{\sin^8 x}+\frac{1}{\cos^2 2x}\right)\left(\sin^8 x+\cos^2 2x\right)=4 \\ \cos^2\sqrt{\frac{\pi^2}{4}-x^2}=1 \end{cases} \quad (32)$$

(32) solution of Equation 2 of the system $x_1 = \frac{\pi}{2}$ va $x_2 = -\frac{\pi}{2}$. Putting these (32) into the first equation of the system, we see that it is the solution of the equation. Say, $x_1 = \frac{\pi}{2}$ and $x_2 = -\frac{\pi}{2}$'s there will be solutions to the given equation.

A: $x_1 = \frac{\pi}{2}$ va $x_2 = -\frac{\pi}{2}$.

12-example. $\lg(\cos x - 0{,}5) + \lg(\sin x - 0{,}3) + 1 = 0$ solve the equation.

Solution. We prove that the given equation does not have a solution. Potens its $(\cos x - 0{,}5)(\sin x - 0{,}3) = \frac{1}{10}$ let's make it look. Based on the inequality between the middle arithmetic and the middle geometric of the left part $\left(ab \leq \frac{1}{4}(a+b)^2\right)$ we appreciate.

$$(\cos x - 0{,}5)(\sin x - 0{,}3) = \left(\frac{\cos x + \sin x - 0{,}8}{2}\right)^2 \leq \left(\frac{\sqrt{2}-0{,}8}{2}\right)^2 <$$

$$< \left(\frac{1{,}42-0{,}8}{2}\right)^2 = (0{,}31)^2 < 0{,}1$$

The left part of the equation is smaller than the right part. Hence, the equation does not have a solution.

A: $\varnothing$.

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