

PACKAGE POSSIBILITIES "MATEMATICA" WHEN SOLVING DIFFERENTIAL EQUATIONS AND SAME- BREED EQUATIONS, THE VARIABLES OF WHICH ARE SEPARATED

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Annotation: The article presents methodological recommendations on the possibilities of studying differential equations, in particular, the benefits of computer mathematical systems. In the study of differential equations, the disclosure of certain aspects of the “matematica” package is considered as the main problem. Examples are given on the capabilities of the “matematica” package when solving differential equations and same-breed equations, the variables of which are separated.

Keywords: differential equation, differential equation in which variables are separated, computer mathematical systems.

Introduction. The use of computer programs significantly reduces the time for performing complex mathematical calculations and presents the results in the desired form (formula, graph, table), which makes it possible to understand the content of the problem and spend more time analyzing the results. The basis for studying the course of differential equations is the study of the main types of differential equations and analytical methods for solving them. Analytical methods in the theory of differential equations are usually understood as methods that allow you to find a clear solution to the issue in the form of a formula that reflects the relationship of the required quantities. Most often, the term "differential equation is solved in quadratures is used. Quadratic solving means expressing as an integral of the combinations of standard functions. The essence of these methods is to determine the type of simple differential equation and solve it according to a previously known algorithm in relation to this type. In our opinion, a prerequisite for studying the course of differential equations is the use of computer programs to solve problems with these methods. In our opinion, as computer-oriented issues, in the process of differential equations, it is necessary to consider issues that require the use of approximate solutions.

Literature review). The problem of organizing the study of differential equations through computer mathematical systems, clarifying its methodological aspects has long been of interest to researchers. For Example, B.S.Gershunsky, V.P. Dyakonov, I.V.Robert, U.X.Khonkulov, V.A.Traynev [1; 2] and others carried out effective methodological research in this direction.

For Example, B.S.Gershunsky defines four directions for the use of computer technology in education. V.P.Dyakonov offers a theoretical, algorithmic development designed to effectively solve all kinds of mathematical problems on computers with a high level of visualization. It is necessary to formulate the content of computer-oriented differential equations, clarify sections, describe a system of measures necessary to carry out the study of differential equations, develop a methodology for studying[3; 4] .

(Research Methodology).

a) *differential equations in which the variables are separated.*

The differential equation, in which the variables are separated, has the following view:

$$y' = f(x)g(y) \quad (1)$$

or

$$f_1(x)g_1(y)dy + f_2(x)f_2(y)dx = 0 \quad (2)$$

The equations of the above (1) and (2) form are separated by variables in the following form and a general solution is found:

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$$\frac{dy}{dx} = f(x)g(y); \quad \frac{dy}{g(y)} = f(x)dx; \quad g(y) \neq 0; \quad \int \frac{dy}{g(y)} = \int f(x)dx + C.$$

In this place $g(y) = 0$ the equation is solved if its true solution $y = a$ if, $y = a$ both (3) the solution to the equation will be. The equation (2) written in differentials is brought to the equation allocated to variables by dividing by a multiple, or $f_1(x)g_2(y) \neq 0$ is charged in the same way as above in the condition.

Example 1. Solve the equation: $y' = \frac{x^2 + 8}{(x^2 - 5x + 6)y^2 \cos y}$

Solution: let's try to solve the equation using the DSolve function.

```

In[1]:= equation = y'[x] ==  $\frac{x^2 + 8}{(x^2 - 5x + 6)y[x]^2 \text{Cos}[y[x] ]}$ 
Out[1]:= y'[x] ==  $\frac{(8 + x^2) \text{Sec}[y[x]]}{(6 - 5x + x^2)y[x]^2}$ 
In[2]:= DSolve[equation, y[x], x]
Solve::tdep : The equations appear to involve the
variables to be solved for in an essentially non-algebraic way. >>
Out[2]:= {{y[x] -> InverseFunction[2 Cos[#1] #1 + Sin[#1] (-2 + #1^2) &][
x + C[1] + 17 Log[-3 + x] - 12 Log[-2 + x]]}}
    
```

However, the DSolve function cannot eat a nonlinear equation. So, we write the equation in the

form of: $y^2 \cos y dy = \frac{x^2 + 8}{(x^2 - 5x + 6)} dx$ we write in the form and integrate both parts:

```

lhs = y^2 Cos[y];
rhs = (x^2 + 8) / (x^2 - 5x + 6);

(*проинтегрируем*)
In[5]:= Integrate[lhs, y]
Out[5]:= 2 y Cos[y] + (-2 + y^2) Sin[y]
In[6]:= Integrate[rhs, x]
Out[6]:= x + 17 Log[-3 + x] - 12 Log[-2 + x]
    
```

The general solution to the equation takes the form:

$2y \cos y + (y^2 - 2) \sin y = x + 17 \log(x - 3) - 12 \log(x - 2) + C$. So after uncomplicated form substitutions, a general solution can be obtained in the Mathematica program.

Example 2. $y(0) = 0$ with the initial condition $\frac{dy}{dx} = \frac{x^2}{\sqrt{9 - x^2} e^y \cos y}$ solve the

equation.

Solution: as in the previous example, we will first clarify the equation and try to solve it using DSolve:

```

In[7]:= equation = y'[x] == x^2 / (Sqrt[9 - x^2] Exp[y[x]] Cos[y[x]])
Out[7]:= y'[x] ==  $\frac{e^{-y[x]} x^2 \text{Sec}[y[x]]}{\sqrt{9 - x^2}}$ 

In[8]:= DSolve[{equation, y[0] == 0}, y[x], x]
Solve::dep : The equations appear to involve the
variables to be solved for in an essentially non-algebraic way. >>
DSolve::bvnul : For some branches of the general solution,
the given boundary conditions lead to an empty solution. >>
Out[8]:= {}
    
```

Say, the use of DSolve is unsuccessful. We divide the variables in the equation. We mark the left and right sides as lhs and rhs, respectively, and integrate them separately:

```

In[9]:= lhs = Exp[y] Cos[y];
In[10]:= rhs = x^2 / Sqrt[9 - x^2];
In[11]:= slhs = Integrate[lhs, y]
Out[11]:=  $\frac{1}{2} e^y (\text{Cos}[y] + \text{Sin}[y])$ 

In[12]:= srhs = Integrate[rhs, x]
Out[12]:=  $-\frac{1}{2} x \sqrt{9 - x^2} + \frac{9}{2} \text{ArcSin}\left[\frac{x}{3}\right]$ 
    
```

General solution of the equation *sol* has the following appearance:

```

In[13]:= sol = slhs == srhs + c
Out[13]:=  $\frac{1}{2} e^y (\text{Cos}[y] + \text{Sin}[y]) = c - \frac{1}{2} x \sqrt{9 - x^2} + \frac{9}{2} \text{ArcSin}\left[\frac{x}{3}\right]$ 
    
```

In addition, it is necessary to find the value *c* with which the initial condition is satisfied. Solve using the function $x = 0, y = 0$ by giving values *c* we find the value:

```

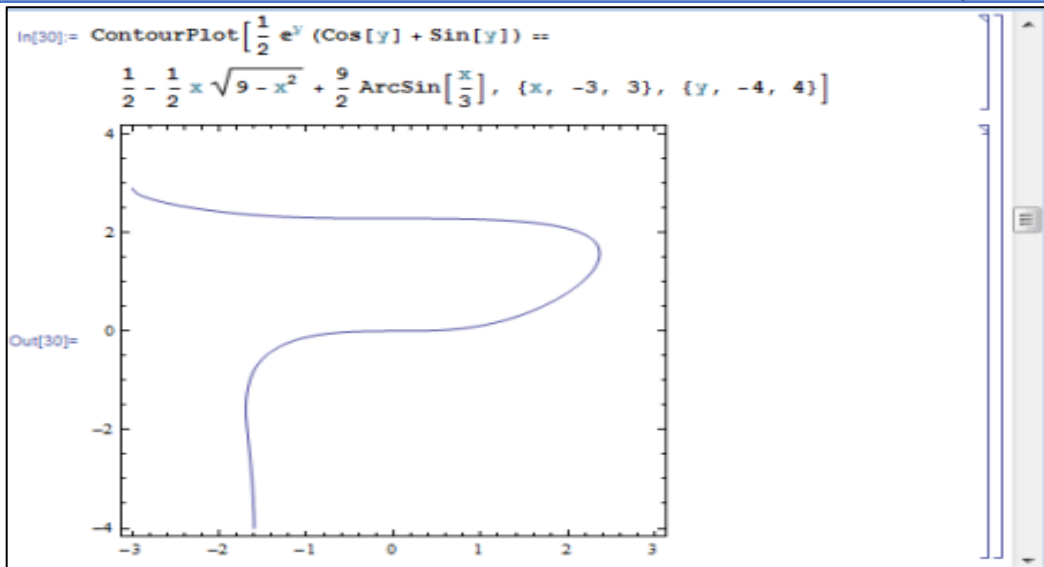
In[14]:= cval = Solve[sol /. y -> 0 /. x -> 0, c]
Out[14]:= {{c ->  $\frac{1}{2}$ }}
    
```

Say solution:

```

In[15]:= solution = sol /. cval[[1]]
Out[15]:=  $\frac{1}{2} e^y (\text{Cos}[y] + \text{Sin}[y]) = \frac{1}{2} - \frac{1}{2} x \sqrt{9 - x^2} + \frac{9}{2} \text{ArcSin}\left[\frac{x}{3}\right]$ 
    
```

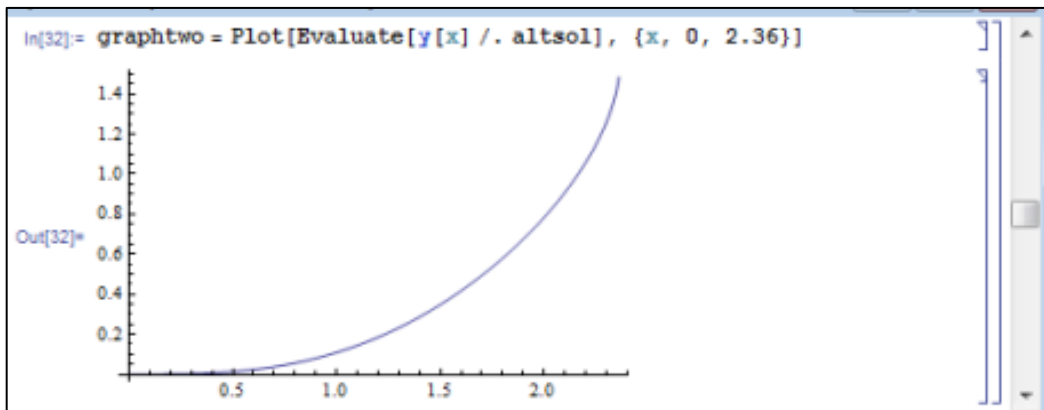
Using the *ContourPlot* function, we describe the graph corresponding to the solution:



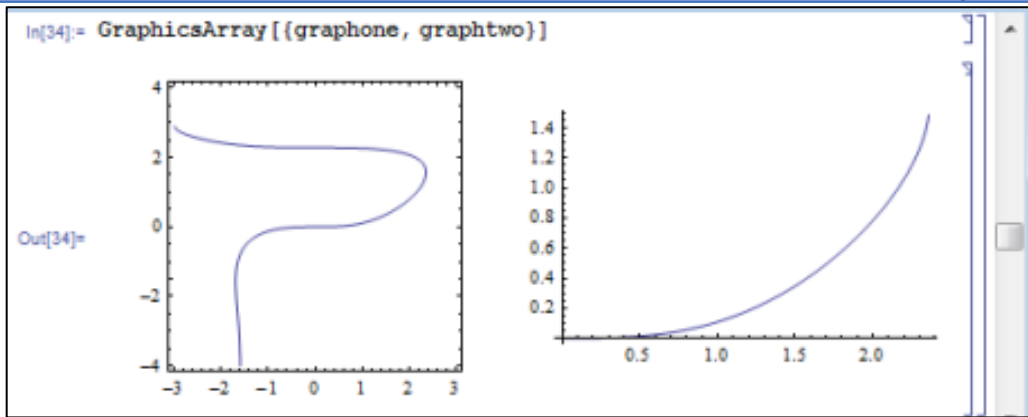
An alternative way to solve an issue is to solve an issue in a given range by number using the NDSolve function. $0 \leq x \leq 2,75$ we use NDSolve to find a numerical solution in the interval:

```
In[31]:= altsol = NDSolve[{equation, y[0] == 0}, y[x], {x, 0, 2.75}]
NDSolve::ndsz: At x == 2.365462613133145, step
size is effectively zero; singularity or stiff system suspected. >>
Out[31]:= {{y[x] -> InterpolatingFunction[{{(0., 2.36546)}, <>][x]}}
```

If $x \leq 2,36$ if NDSolve a solution can be found using the function:



We use the GraphicsArray function to compare both solutions at the same time:



b) Same- breed and full differential equations.
This

$$y' = \varphi\left(\frac{y}{x}\right) \quad (3)$$

the equation in appearance is called the same-breed differential equation, where φ - given function. To solve this equation, usually, $y = ux$ replacement will be performed $y = ux$ and $y' = u'x + u$ putting the expressions (3) in the equation, $u' = (\varphi(u) - u) / x$ we form an equation in which the variables in the form are separated. $f(x, y)$ let the function be given. If an optional real number $\lambda > 0$ for such m real number, $f(\lambda x, \lambda y) = \lambda^m f(x, y)$ when equality is fulfilled, then $f(x, y)$ function m - an ordered same-sex function is called. If $M(x, y)$ and $N(x, y)$ if the functions are same-order same-sex functions, then

$$M(x, y)dx + N(x, y)dy = 0 \quad (4)$$

the equation will be a same-sex differential equation. This equation is also $y = ux$ with the help of substitution, the variable is solved by causing it to a separable equation.

Explanation. This $y' = f\left(\frac{a_1x + b_1y + c_1}{a_2x + b_2y + c_2}\right)$, $a_i, b_i, c_i = const, i = 1, 2$

is brought to the same-sex equation by some substitution.

Example 1. Solve the equation: $(x + y)dx - xdy = 0$.

Solution: $N(x, y) = -x$ and $M(x, y) = x + y$ we enter the waistband. Because $M(tx, ty) = (tx) + (ty) = t(x + y) = tM(x, y)$ and $N(tx, ty) = -tx = tN(x, y)$ from the fact that $(x + y)dx - xdy = 0$ the first degree is a same-sex equation.

We get a general solution using the DSolve function of the equation:

```

In[5]= DSolve[(x + y[x]) Dt[x] - xDt[y[x]] == 0, y[x], x]
Out[5]= {{y[x] -> x C[1] + x Log[x]}}
    
```

Example 2. Solve the equation: $(x^{\frac{1}{3}}y^{\frac{2}{3}} + x)dx + (x^{\frac{2}{3}}y^{\frac{1}{3}} + y)dy = 0$.

Solution: $capm[x, y] = x^{\frac{1}{3}} y^{\frac{2}{3}} + x$ and $capn[x, y] = x^{\frac{2}{3}} y^{\frac{1}{3}} + y$ we enter the marks. We check that the equation is homogeneous:

```

In[1]:= capm[x_, y_] = x^(1/3) y^(2/3) + x;
In[2]:= capn[x_, y_] = x^(2/3) y^(1/3) + y;
(*проверим однородность capm*)
In[3]:= capm[t x, t y]
Out[3]:= t x + (t x)^(1/3) (t y)^(2/3)
(*соберем степени*)
In[4]:= stepone = PowerExpand[capm[t x, t y]]
Out[4]:= t x + t x^(1/3) y^(2/3)
    
```

Collect using the function, t we raise the ni from the stepone expression to a certain level:

```

steptwo = Collect[stepone, t]
t (x + x^(1/3) y^(2/3))
    
```

capn we repeat the above steps to mark:

```

(*аналогично проверим однородность capn*)
stepthree = PowerExpand[capn[t x, t y]]
t x^(2/3) y^(1/3) + t y
stepfour = Collect[stepthree, t]
t (x^(2/3) y^(1/3) + y)
    
```

Thus, the initial equation is the same-sex equation of the 1st degree. We use the DSolve function to solve the equation:

```

In[7]:= sols = DSolve[capm[x, y[x]] + capn[x, y[x]] y'[x] == 0, y[x], x]
Out[7]:= {{y[x] -> -sqrt(-x^(2/3) x^(2/2))}, {y[x] -> (-3 x^(4/3) + 4 C[1])^(3/4) / 3^(3/4)}}
    
```

As a result, we get two solutions. However, we can solve a given equation using a standard algorithm for solving same-sex equations, that is, it is possible to substitute variables and bring the equation into an equation in which the variables are separated. We implement this algorithm in the Mathematica package. We determine the left side of the equation. As you know $Dt[x]$ function dx to and $Dt[y]$ while dy corresponds to:

```

In[7]:= Clear[x, y, u]
In[8]:= leqone = capm[x, y] Dt[x] + capn[x, y] Dt[y]
Out[8]:= (x + x^(1/3) (u x)^(2/3)) Dt[x] + (u x + x^(2/3) (u x)^(1/3)) Dt[y]
    
```


$y = ux$ let's put the definition, here u New required function:

```
In[9]:= y = u x
Out[9]= u x

In[10]:= leqtwo = leqone // PowerExpand // ExpandAll
Out[10]= u1/3 x2 Dt[u] + u x2 Dt[u] + x Dt[x] + u2/3 x Dt[x] + u4/3 x Dt[x] + u2 x Dt[x]
```

Terms and conditions $Dt[x]$, $Dt[u]$ Express with:

```
In[11]:= leqthree = Collect[leqtwo, {Dt[x], Dt[u], x, u}]
Out[11]= (u1/3 + u) x2 Dt[u] + (1 + u2/3 + u4/3 + u2) x Dt[x]
```

We can solve the equation as an equation in which the variables are separated. To do this, we select the parts of the equation that need to be divided to separate the variables:

```
leqthree[[1, 2]] (*первая часть уравнения, второй множитель*)
Out[12]= x2

In[13]:= leqthree[[2, 1]] (*вторая часть уравнения, второй множитель*)
Out[13]= 1 + u2/3 + u4/3 + u2
```

We separate the variables:

```
In[14]:= leqfour = Cancel[Apart[leqthree / (leqthree[[1, 2]] leqthree[[2, 1]])]]
Out[14]=  $\frac{u^{1/3} Dt[u] + Dt[x]}{1 + u^{4/3}} + \frac{Dt[x]}{x}$ 
```

If we equate this expression to zero, it will be the equation in which the variables of the equation

are separated: $\frac{u^{1/3} Dt[u]}{1 + u^{4/3}} = -\frac{Dt[x]}{x}$. We integrate both parts of the resulting expression. We write the

solution of the initial equation in the natila of the alternation of variables:

```
In[18]:= u = y / x
Out[18]=  $\frac{y}{x}$ 

In[19]:= first = -second + constant
Out[19]=  $\frac{3}{4} \text{Log}\left[1 + \left(\frac{y}{x}\right)^{4/3}\right] = \text{constant} - \text{Log}[x]$ 

In[20]:= solution = Exp[first] = const Exp[-second] // Simplify
Out[20]=  $\left(1 + \left(\frac{y}{x}\right)^{4/3}\right)^{3/4} = \frac{\text{const}}{x}$ 
```

Thus, it turns out that the general solution of a given equation is in the form of:

$$\left(1 + \left(\frac{y}{x}\right)^{\frac{4}{3}}\right)^{\frac{3}{4}} = \frac{c}{x}, \text{ in this place } c \text{ optional invariant number.}$$

(Conclusion/Recommendations). When using computer programs to obtain an analytical solution of a differential equation, students receive a ready-made answer in a symbolic form and cannot study a solution algorithm. At the same time, when solving a differential equation using computer programs, it is not necessary to know what type it belongs to. Since the study of the types of differential equations and methods of their solution is the basis of the course, it seems inappropriate for us to use computer programs to find an analytical solution. But at the same time, the main disadvantage of analytical methods is that most differential equations do not belong to certain types, so their solution cannot be obtained using analytical methods. Therefore, students will also need to learn about approximate methods of solving differential equations.

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