CONSTRUCTION OF A GENERAL SOLUTION OF ONE CLASS OF LINEAR DIFFERENCE EQUATIONS OF ORDER n

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ANNOTATION: In this paper we study the construction of a general solution to one class of difference equations of the following form

$x(t+n)+a_1(t)x(t+n-1)+\cdots+a_n(t)x(t)=f(t),$

where t are known functions of variable t, known functions of variable t, unknown function. $\in R=(-\infty,+\infty),\ a_i(t), i=1,\ldots n a_n(t)\neq 0, t\in R), f(t)-x(t)-$

Keywords: general, difference, solution, particular solution, general solution, difference equation.

Consider an equation of the form

$(1) x(t+n)+a_1(t)x(t+n-1)+\cdots+a_n(t)x(t)=f(t)\,,$

where , , are known functions of the variable , the unknown function is called a linear inhomogeneous difference equation of order.$t\in R=(-\infty,+\infty) a_i(t),\ i=1\,,\ldots\,, n f(t)-t x(t)-n-$

Using the method of sequential integration, it is possible, as in the case of a homogeneous equation [3], to construct various kinds of partial solutions to equation (1).

Theorem. Let there be some solution to equation (1). Then the general solution of the inhomogeneous equation (1) is the sum of the general solution of the corresponding homogeneous equation (1) and .$\gamma(t)-\ \gamma(t)$

Proof. Because

$$\gamma(t+n)+a_1(t)\gamma(t+n-1)+\cdots+a_n(t)\gamma(t)\equiv f(t)\,,$$

then assuming in (1)

$$x(t)=y(t)+\gamma(t),\ (2)$$

we get

$$y(t+n)+a_1(t)y(t+n-1)+\cdots+a_n(t)y(t)+$$
$$+\gamma(t+n)+a_1(t)\gamma(t+n-1)+\cdots+a_n(t)\gamma(t)\equiv f(t),$$

It follows that the function is indeed a solution to equation (1).$x(t)=y(t)+\gamma(t)$

Let us now show that function (2) is a general solutionequation (1). To do this, take any solution to equation (1) and consider the difference$x(t)$

$x(t)-\ \gamma(t)$. This difference is the solution to the corresponding homogeneous equation

. $(3) y(t+n)+a_1(t)y(t+n-1)+\cdots+a_n(t)y(t)=0$

Really,

ISSN 2277-3630 (online), Published by International journal of Social Sciences & Interdisciplinary Research., under Volume: 13 Issue: 04 in April-2024
https://www.gejournal.net/index.php/IJSSIR

$$x(t+n) - \gamma(t+n) + a_1(t)[x(t+n-1) - \gamma(t+n-1)] + \cdots + a_n(t)[x(t) - \gamma(t)]$$
$$= x(t+n) - \gamma(t+n) +$$
$$+a_1(t)x(t+n-1) - a_1(t)\gamma(t+n-1) + \cdots + a_n(t)x(t) -$$
$$-a_n(t)\gamma(t) = x(t+n) + a_1(t)x(t+n-1) + \cdots + a_n(t)x(t) -$$
$$-[\gamma(t+n) + a_1(t)\gamma(t+n-1) + \cdots + a_n(t)\gamma(t)] = f(t) - f(t) = 0$$

This means that the difference can be written as$x(t) - \gamma(t)$

,$x(t) - \gamma(t) = \sum_{i=1}^{n} \omega_i^0(t) y_i(t)$

Where

, (4)$x(t) = \gamma(t) + \sum_{i=1}^{n} \omega_i^0(t) y_i(t)$

where , determination of the value of periodic period 1 functions. So, any solution to equation (1) is obtained from formula (2) with the appropriate selection of arbitrary periodic functions , , of period 1, i.e., function (2) is a general solution to equation (1).$\omega_i^0(t) i = 1, \dots, n - x(t)\ \omega_i^0(t) i = 1, \dots, n$

Thus, the problem of constructing a general solution to the inhomogeneous equation (1) is reduced to constructing a general solution to the homogeneous equation (3), i.e., to find a general solution to the linear inhomogeneous equation (1), you need to find a general solution to the corresponding homogeneous equation and some particular solution of a non-homogeneous equation.

The theorem has been proven.

Let us now consider one class of linear homogeneous difference equations of order for which a general solution can be constructed. Namely, consider the equation $n -$

$$x(t+n) + a_1(t)x(t+n-1) + \cdots + a_n(t)x(t) = 0 (5)$$

under the assumption that the functions , are continuous and periodic. Then, to construct a general continuous solution, it is sufficient, according to Theorem 1 in [3], to find particular continuous solutions for which the condition is satisfied$a_i(t) i = 1, \dots, n, 1 - n x_i(t), i = 1, \dots, n$,

$$. (6) \mathcal{W}(t) = \begin{vmatrix} x_1(t) & x_2(t) & \dots & x_n(t) \\ x_1(t+1) & x_2(t+1) & \dots & x_n(t+1) \\ \dots & \dots & \dots & \dots \\ x_1(t+n-1) & x_2(t+n-1) & \dots & x_n(t+n-1) \end{vmatrix} \neq 0$$

We construct these solutions as follows.

Substituting into (5) the expression

$$x(t) = \lambda^t(t) (7)$$

where) is some as yet undefined continuous and is a periodic function$\lambda(t 1$

() for) , we get $\lambda(t \neq 0 t \in R$

$$\lambda^t(t)[\lambda^n(t+n) + a_1(t)\lambda^{n-1}(t) + \cdots + a_{n-1}(t)\lambda(t) + a_n(t)] = 0$$

Since), it follows from the last relation that the function will be a solution to equation (5) only in the case when the function) satisfies the characteristic equation$\lambda(t \neq 0 \lambda^t(t) \lambda(t$

$$\lambda^n(t+n) + a_1(t)\lambda^{n-1}(t) + \cdots + a_{n-1}(t)\lambda(t) + a_n(t) = 0. (8)$$

Let us denote , the roots of equation (8) and assume , , t. (Note that since the roots of equation (8) continuously depend on the coefficients , and the functions are continuous and periodic, then the functions are also continuous and periodic). Then, by virtue of (7), each , corresponds to a solution$\lambda_i(t) i = 1, \dots, n - \lambda_i(t) \neq \lambda_j(t) i, j = 1, \dots, n \in R\ a_i(t),\ i = 1, \dots, n\ 1 \lambda_i(t), i = 1, \dots, n 1 \lambda_i(t), i = 1, \dots, n$

,$x_i(t) = \lambda_i^t(t) i = 1, \dots, n$.

(9)

Let's show thatfunction (9) satisfies the condition

ISSN 2277-3630 (online), Published by International journal of Social Sciences & Interdisciplinary Research., under Volume: 13 Issue: 04 in April-2024
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$$\mathcal{W}(t) = \begin{vmatrix} x_1(t) & x_2(t) & \dots & x_n(t) \\ x_1(t+1) & x_2(t+1) & \dots & x_n(t+1) \\ \dots & \dots & \dots & \dots \\ x_1(t+n-1) & x_2(t+n-1) & \dots & x_n(t+n-1) \end{vmatrix} \neq 0,$$

i.e., condition (6).

Indeed, since

$$\mathcal{W}(t) = \begin{vmatrix} \lambda_1^t(t) & \lambda_2^t(t) & \dots & \lambda_n^t(t) \\ \lambda_1^{t+1}(t) & \lambda_2^{t+1}(t) & \dots & \lambda_n^{t+1} \\ \dots & \dots & \dots & \dots \\ \lambda_1^{t+n-1}(t) & \lambda_2^{t+n-1}(t) & \dots & \lambda_n^{t+n-1}(t) \end{vmatrix} == [\lambda_1(t) \dots \lambda_n(t)]^t \begin{vmatrix} 1 & 1 & \dots & 1 \\ \lambda_1(t) & \lambda_2(t) & \dots & \lambda_n(t) \\ \dots & \dots & \dots & \dots \\ \lambda_1^{n-1}(t) & \lambda_2^{n-1}(t) & \dots & \lambda_n^{n-1}(t) \end{vmatrix} =$$

$$= [\lambda_1(t) \dots \lambda_n(t)]^t \prod_{1 \le j < i \le n} (\lambda_i(t) - \lambda_j(t)),$$

then when $\mathcal{W}(t) \neq 0 t \in R$.From here, according to Theorem 2 in [3], the general solution of equation (5) has the form

,$x(t) = \sum_{i=1}^{n} \lambda_i^t(t) \omega_i(t)$

where arbitrary continuous are periodic functions. $\omega_i(t), i = 1, \dots, n-1$

If amongroots are equal, then it is easy to show that the corresponding solutions (9) do not satisfy the condition$\lambda_1(t) \dots \lambda_n(t)$

$$\mathcal{W}(t) = \begin{vmatrix} x_1(t) & x_2(t) & \dots & x_n(t) \\ x_1(t+1) & x_2(t+1) & \dots & x_n(t+1) \\ \dots & \dots & \dots & \dots \\ x_1(t+n-1) & x_2(t+n-1) & \dots & x_n(t+n-1) \end{vmatrix} \neq 0,$$

i.e., condition (6).

In this case, partial solutions satisfying condition (6) have the following form$n$ $x_1(t), x_2(t), \dots, x_n(t)$

$\lambda_i^t(t),\ t\lambda_i^t(t), \dots,, t^{p_i - 1}\lambda_i^t(t) i = 1, \dots, k < n,$

where , , are the multiplicities of roots , . This can be proven in a manner similar to how it was done in the first case.$p_i i = 1, \dots, k - \lambda_i(t) i = 1, \dots, k, p_1 + p_2 + \dots + +p_k = n$

Therefore, in this casethe general continuous solution of equation (5) has the form

$$x(t) = \lambda_1^t(t)\omega_1^1(t) + t\lambda_1^t(t)\omega_2^1(t) + \dots + t^{p_1-1}\lambda_1^t(t)\omega_{p_1}^1(t) + \dots + \lambda_k^t(t)\omega_1^k(t) +$$
$$+ \dots + t^{p_k-1}\lambda_k^t(t)\omega_{p_k}^k(t),$$

where , , are arbitrary continuous – periodic functions.$\omega_1^i(t) \ \dots, \omega_{p_i}^i(t) i = 1, \dots, k, \ - \ 1.$

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ISSN 2277-3630 (online), Published by International journal of Social Sciences & Interdisciplinary Research., under Volume: 13 Issue: 04 in April-2024
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