RESULTS OF THE MINIMAX PRINCIPLE FOR SELF-ADJOINT OPERATORS IN THE FRIEDRICHS MODEL

Davron Kulturayev Jurayevich
senior teacher, Termiz State Pedagogical Institute
Uzbekistan, Termiz city, Islam Karimov street, 288b
Email:davron_2189@mail.ru
Bakhriddin Oltiyev Jurayevich
teacher, Termiz State Pedagogical Institute
Uzbekistan, Termiz city, Islam Karimov street, 288b
Email:oltiyevbakhriddin@gmail.com

Abstract: The discrete spectrum of self-adjoint operators in the Friedrichs model is studied. A sufficient condition for the existence of an infinite number of eigenvalues of the Friedrichs model is given. It is proved that the negative eigenvalues of the discrete Schrödinger operator are infinite.

Key words: Friedrichs model, self-adjoint operator, spectrum, critical spectrum, discrete spectrum, non-separated kernel.

Introduction

Let's assume u(x) function $\Omega_\nu = [0,1]^\nu, \nu \in \mathbb{N}$ be a real-valued non-negative continuous function on the set and take zero value, i.e $0 \in Ran(u)$. K - operator $L_2(\Omega_\nu)$ in Gilbert space $k(x,s) \in L_2(\Omega_\nu^2)$ be a compact integral operator with a symmetric kernel. Some issues of quantum mechanics and statistical physics (see [1-3]). $L_2(\Omega_\nu)$ in hilbert space the following

$$H = H_0 - K, \quad (1.1)$$

determined by the formula H is brought to study the spectrum of the operator, here

$$(H_0 f)(x) = u(x) f(x), \ \ (Kf)(x) = \int_{\Omega_\nu} k(x,s) f(s) d\mu(s).$$

In this K is a compact integral operator integral in the sense of Lebesgue and $\mu(\cdot)$ - an expression $\mathbb{R}^\nu$ is a Lebesgue measure on the set. From Weyl's theorem [4] about compact excitation H of the operator $\sigma_{ees}(H)$ important spectrum $u(x)$ it follows that the function consists of a set of values, i.e $\sigma_{ess}(H) = [0, u_{\max}]$ equality is appropriate, here $u_{\max} = \max_{x \in \Omega_\nu} u(x)$.

The operator in the form (1.1) is called the operator in the Friedrichs model. Operators in the Friedrichs model were studied in works [5-8, 11-14].

$H = H_0 - K$ discrete spectrum of the operator $I - K_\lambda$ of the operator $\Delta(\lambda), \lambda \in \mathbb{C}$, $Ran(u)$ [17] The Fredholm determinant overlaps with zeros, here I - unit operator and $K_\lambda = K(H_0 - \lambda I)^{-1}, \lambda \in \mathbb{C}$, $Ran(u)$.

of the Fredholm determinant

$$\Delta(\lambda) = 0, \lambda \in \mathbb{C} \ , \ Ran(u) \quad (1.2)$$

the study of the number of zeros is the main issue of the theory of operators in the Friedrichs model.

If the following

ISSN 2277-3630 (online), Published by International journal of Social Sciences & Interdisciplinary Research., under Volume: 13 Issue: 04 in April-2024
https://www.gejournal.net/index.php/IJSSIR
Copyright (c) 2024 Author (s). This is an open-access article distributed under the terms of Creative Commons Attribution License (CC BY). To view a copy of this license, visit https://creativecommons.org/licenses/by/4.0/

$$\lim_{\xi \to 0-0} \int_{\Omega_\nu} \frac{dx}{u(x)-\lambda}, \lambda < 0$$

is sufficiently small if limit has a finite value $\varepsilon > 0$ for number [5] $\sigma(H_0 - \varepsilon K) = \sigma(H_0) = [0, u_{max}]$ equality is reasonable, i.e. small enough $\varepsilon > 0$ at $H_\varepsilon = H_0 - \varepsilon K$ the discrete spectrum of the operator will not exist.

Using minimax and maximin principles [6], if K if the kernel of the integral operator is separated, then the discrete spectrum of the Friedrichs model (1.1) is finite. It follows from (1.1) that the discrete spectrum of the operator in the Friedrichs model is infinite K it follows that the kernel of the integral operator is in a non-separable form. The question of the infinity of eigenvalues outside the critical range of the one-dimensional model (1.1) was studied in works [6,7]. [8] considered the question of the existence of infinitely many eigenvalues of the multidimensional Friedrichs model and found necessary and sufficient conditions for the infiniteness of the discrete spectrum.

In this review, we describe a method for studying the infinity of the discrete spectrum of operators in the Friedrichs model. The second section presents some auxiliary concepts and definitions derived from the minimax principle. In the third section (1.1), one criterion is proved, which ensures that the negative eigenvalues of the Friedrichs model are infinite. The fourth section shows that the discrete spectrum of one discrete Schrödinger operator is infinite.

2. Several auxiliary concepts and definitions

Let's assume H - separable hilbert space and $A : \mathrm{H} \to \mathrm{H}$ let the linearly bounded element be given an adjoint operator.

For convenience $\sigma(A)$, $\sigma_{ess}(A)$ and $\sigma_{disc}(A)$ with A we denote the spectrum, critical spectrum and discrete spectrum of the operator, respectively [16].

We also include the following definitions

$$E_{min}(A) = inf\{\lambda : \lambda \in \sigma_{ess}(A)\},\ E_{max}(A) = sup\{\lambda : \lambda \in \sigma_{ess}(A)\}.$$

Here $E_{min}(A)$ ($E_{max}(A)$ number) number A is called the lower (upper) limit of the critical spectrum of the operator.

If optional $x \in \mathrm{H}$ for $(Ax, x) \geq 0$ if , then is linearly bounded integral A operator is called a positive operator and $A \geq 0$ or $0 \leq A$ is written in the form

(1.1) in the Friedrichs model H_0 and K the following properties are relevant for operators. H_0 operator is optional to be positive $x \in \Omega_\nu$ at $u(x) \geq 0$ the fulfillment of inequality is necessary and sufficient. K and for the integral operator to be positive, it is necessary and sufficient that each of its eigenvalues be negative.

$\{\mu_n(A)\}_{n\in\square}$ join with A for the operator we define a bounded sequence of increasing real numbers constructed using the minimax principle [6]. Then each $\mu_n(A),\ n \in \square$ number A will be an eigenvalue of the operator and $\lim_{n\to\infty} \mu_n(A) = E_{min}(A)$ equality is appropriate.

Lemma 2.1.[6] $A, B : \mathrm{H} \to \mathrm{H}$ - linearly bounded adjoint operators and $A \leq B$ let the inequality be appropriate. In that case $\mu_n(A) \leq \mu_n(B),\ n \in \square$ Inequality is relevant here

$$\mu_k(A) = \sup_{L \subset \mathrm{H},\ dimL = k-1} \ \inf_{\|x\|=1,\ x \perp L} (Ax, x),\ k \in \square .$$

The following assertion follows from Lemma 2.1.

ISSN 2277-3630 (online), Published by International journal of Social Sciences & Interdisciplinary Research., under Volume: 13 Issue: 04 in April-2024
https://www.gejournal.net/index.php/IJSSIR

Copyright (c) 2024 Author (s). This is an open-access article distributed under the terms of Creative Commons Attribution License (CC BY). To view a copy of this license, visit https://creativecommons.org/licenses/by/4.0/

Confirmation 2.1. H in hilbert space A and B for linear bounded adjoint operators $A \le B$ inequality and $E_{min}(A) = E_{min}(B)$ let equality be appropriate. If B If the discrete spectrum lying on the left side of the lower limit of the essential spectrum of the operator is infinite, then A The discrete spectrum lying to the left of the lower limit of the critical spectrum of the operator is also infinite.

Suppose (1.1) is in the Friedrichs model K the integral operator is positive and K let the kernel of the integral operator be in non-separated form, i.e

$$k(x,s) = \sum_{n=1}^{\infty} a_n \varphi_n(x)\overline{\varphi_n(s)},$$

here $\{a_k\}_{k\in\mathbb{N}} \in l_2, a_k > 0,\ k \in \mathbb{N}$, each a_k number K the eigenvalue of the operator and $\{\varphi_k(x)\}_{k\in\mathbb{N}}$ - system K of the operator $L_2(\Omega_\nu)$ in hilbert space a_k is a system of orthonormal eigenfunctions corresponding to the eigenvalues of

K that the operator is positive $K^{\frac{1}{2}}$ has a square root and

$$K^{\frac{1}{2}} f(x) = \int_{\Omega_\nu} k^{\frac{1}{2}}(x,s) f(s)ds, \text{ bu yerda } k^{\frac{1}{2}}(x,s) = \sum_{n=1}^{\infty} a_n^{\frac{1}{2}} \varphi_n(x)\overline{\varphi_n(s)}$$

appears [10]. H_0 and for the operator $H_0^{\frac{1}{2}} f(x) = u^{\frac{1}{2}}(x) f(x)$ equality is appropriate.
also H (1.1) from the fact that the operator is self-adjoint $\sigma(H) \subset \mathbb{R}$ relationship arises. K and from the positivity of the operator $\sigma(H) \cap (u_{\max}, \infty) = \varnothing$ it follows that the equality holds, therefore H discrete spectrum of the operator $(-\infty, 0)$ lies in between.

Confirmation 2.2. H (1.1) Let the operator in the Friedrichs model have an infinite number of negative eigenvalues. If negative continuous $v(x),\ 0 \in Ran(v)$ function has a multiplication operator V_0 for the operator $H_0 \ge V_0$ if the condition is met, then

$$H_1 = V_0 - K$$

operator also has an infinite number of negative eigenvalues.

Proof. let's say $H_0 \ge V_0$ let the inequality be appropriate. Then it's optional $f(x) \in L_2(\Omega_\nu)$ for $((H_0 - V_0) f, f) \ge 0$ the inequality holds. From this

$$0 \le ((H_0 - V_0) f, f) = ((H_0 - K + K - V_0) f, f) =$$
$$= (((H_0 - K) - (V_0 - K)) f, f) = ((H - H_1) f, f)$$

it follows that the relationship is appropriate. As a result $H \ge H_1$ the inequality becomes relevant. From this and Proposition 2.1 H_1 operator has infinitely many negative eigenvalues.

Confirmation 2.3. H (1.1) Let the operator in the Friedrichs model have an infinite number of negative eigenvalues. If $q(x,s)$ uninucleate Q for a positive compact integral operator $Q \ge K$ if the condition is appropriate then

$$H_2 = H_0 - Q$$

operator also has an infinite number of negative eigenvalues.
The proof of Proposition 2.3 is proved similar to the proof of Proposition 2.2.

We consider the operator in the Friedrichs model as follows

$$H_3 = V_0 - Q,$$

ISSN 2277-3630 (online), Published by International journal of Social Sciences & Interdisciplinary Research., under Volume: 13 Issue: 04 in April-2024
https://www.gejournal.net/index.php/IJSSIR
Copyright (c) 2024 Author (s). This is an open-access article distributed under the terms of Creative Commons Attribution License (CC BY). To view a copy of this license, visit https://creativecommons.org/licenses/by/4.0/

here V_0 - continuous negative $v(x), 0 \in Ran(v)$ function multiplication operator, Q - not separated $q(x,s) = \sum_{n=1}^{\infty} b_n \psi_n(x)\overline{\psi_n(s)},\ b_n > 0,\ n \in \mathbb{N}$ is a positive compact integral operator with a kernel.

Theorem 2.1. H (1.1) Let the operator in the Friedrichs model have an infinite number of negative eigenvalues. If $H_0 \geq V_0$ and $Q \geq K$ if the conditions are met, then $H_3 = V_0 - Q$ operator also has an infinite number of negative eigenvalues.

*Proof.*Let's assume $H_0 \geq V_0$ and $Q \geq K$ let the inequalities be appropriate. Then it's optional by definition $f(x) \in L_2(\Omega_\nu)$ for $0 \leq ((H_0 - V_0) f, f)$ and $0 \leq ((Q-K) f, f)$ inequalities are relevant. From this

$$0 \leq ((H_0 - V_0) f, f) + ((Q-K) f, f) = (((H_0 - V_0) + (Q-K)) f, f) =$$
$$= (((H_0 - K) - (V_0 - Q)) f, f) = ((H - H_3) f, f),\ f(x) \in L_2(\Omega_\nu)$$

attitude is appropriate. As a result $H \geq H_3$ inequality arises.

On the other hand $E_{min}(H) = E_{min}(H_3) = 0$ equality is appropriate. From this and Proposition 2.1 H_3 it follows that the operator has infinitely many negative eigenvalues.

3. About the sign of infinity of negative eigenvalues of the Friedrichs model

in the Friedrichs model H the operator $L_2(\Omega_\nu \times \Omega_\nu)$ in the hilbert space we look as follows

$$H = H_0 - K, \quad (3.1)$$

here

$$(H_0 f)(x,y) = u(x,y) f(x,y),\ (Kf)(x,y) = \int_{\Omega_\nu}\int_{\Omega_\nu} k(x,y;s,t) f(s,t) d\mu(s) d\mu(t).$$

In this $u(x,y) \in C(\Omega_\nu, \Omega_\nu)$ the function is nonnegative and $0 \in Ran(u)$, $k(x,y;s,t) \in L_2(\Omega_\nu^2 \times \Omega_\nu^2)$ and the core is symmetrical, i.e $k(x,y;s,t) = \overline{k(s,t;x,y)}$.

let's say K operator is infinite $\eta_1 > \eta_2 > \ldots > \eta_n > \ldots,\ \eta_n \to 0,\ n \to \infty$ has positive eigenvalues and $\{g_n(x,y)\}_{n \in \mathbb{N}}$ system K let the operator be a sequence of orthonormal eigenfunctions corresponding to these eigenvalues.

Optional $\xi < 0$ we define the following integral operators for

$$P(\xi) = K^{\frac{1}{2}} r_0(\xi) K^{\frac{1}{2}},\ R(\xi) = K^{\frac{1}{2}} r_0^{\frac{1}{2}}(\xi),$$

here $r_0(\xi)$ - H_0 the resolvent of the operator. $P(\xi) = R(\xi)(R(\xi))^*$ from equality $P(\xi)$ it follows that the operator is positive. $Hf = \xi f$ of Eq f_0 with the solution $P(\xi)$ of the operator φ The fixed point is connected by the following relations.

$$f_0 = r_0(\xi) K^{\frac{1}{2}} \varphi,\ \varphi = K^{\frac{1}{2}} f_0. \quad (3.2)$$

Lemma 3.1.[9] $\xi < 0$ number H to be an eigenvalue of the operator $\lambda = 1$ number $P(\xi)$ it is necessary and sufficient that the operator has an eigenvalue.

From Lemma 3.1 $dimKer(H - \xi I) = dimKer(P(\xi) - I),\ \xi < 0$ equality follows.

We introduce the following definition

$$\Phi(\xi) = \int_{\Omega_\nu}\int_{\Omega_\nu} \frac{dxdy}{u(x,y) - \xi},\ \xi < 0.$$

ISSN 2277-3630 (online), Published by International journal of Social Sciences & Interdisciplinary Research., under Volume: 13 Issue: 04 in April-2024
https://www.gejournal.net/index.php/IJSSIR

Copyright (c) 2024 Author (s). This is an open-access article distributed under the terms of Creative Commons Attribution License (CC BY). To view a copy of this license, visit https://creativecommons.org/licenses/by/4.0/

Theorem 3.1. Assume that (3.1) is in the Friedrichs model $u(x,y)$ and $k(x,y;s,t)$ for functions $u(x,y)=u_0(y)$ and $k(x,y;s,t)=k_0(x,s)$ let the equalities be appropriate. If $\lim_{\xi\to 0-0}\Phi(\xi)=+\infty$ if the condition is appropriate then H (3.1) operator is infinite negative $\xi_n,\ n\in\mathbb{N}$ have eigenvalues and the eigenfunctions corresponding to these eigenvalues look like this

$$f_n(x,y)=\frac{g_n^0(x)}{u_0(y)-\xi_n}\ (3.3)$$

here $x=(x_1,x_2,...x_\nu),\ y=(y_1,y_2,...,y_\nu)$.

Proof. $\xi<0$ being $P(\xi)$ of the operator $p(\xi,x;z)$ for the core $p(\xi;x,z)=\Phi(\xi)k_0(x,z)$ equality is appropriate. From this

$$P(\xi)=\Phi(\xi)K\ (3.4)$$

equality follows. This is equality K characteristic function of the operator $P(\xi)$ means that it is also a characteristic function of the operator.

From the condition of Theorem 3.1 $\eta_n,n\in\mathbb{N}$ numbers K nonzero eigenvalues of the operator, $g_n(x,y)=g_n^0(x)\in L_2(\Omega_\nu),\ n\in\mathbb{N}$ and functions K of the operator $\eta_n,n\in\mathbb{N}$ will be eigenfunctions corresponding to eigenvalues, i.e $L_2(\Omega_\nu)$ is orthonormal in the Hilbert space $\{g_n^0\}_{n\in\mathbb{N}}$ there is a sequence of functions such that $k(x,s)=\sum_{n=1}^{\infty}\eta_n g_n(x)\overline{g_n(s)}$ equality is appropriate.

Then from equality (3.4).

$$\lambda_n(\xi)=\eta_n\Phi(\xi),\ \ n\in\mathbb{N}\ (3.5)$$

numbers $P(\xi)$ it follows that the operator has eigenvalues.

Now each $n\in\mathbb{N}$ for the following

$$\lambda_n(\xi)=1\,(3.6)$$

the equation is negative $\xi_n<0$ we show that it has a solution. For this, (3.5) is equivalent to equation (3.6) taking into account the equality $\Phi(\xi)=\frac{1}{\eta_n}$ we get the equation The following $\Phi'(\xi)=\int_{\Omega_\nu}\frac{dy}{(u_0(y)-\xi)^2}$ derivative $(-\infty,0)$ since it is positive in the interval $\Phi(\xi)$ function $(-\infty,0)$ is increasing and positive in the interval. Besides $\Phi(\xi)$ for the function $\lim_{\xi\to 0-0}\Phi(\xi)=+\infty$ and $\lim_{\xi\to-\infty}\Phi(\xi)=0$ relationships are appropriate.

From this, equation (3.6) is assigned to each $n\in\mathbb{N}$ negative for ξ_n it follows that it has a solution. From Lemma 3.1, each $\xi_n,\ n\in\mathbb{N}$ number H will be an eigenvalue of the operator. $P(\xi)$ operator $g_n(x,y)=g_n^0(x_1,x_2,...,x_\nu),\ n\in\mathbb{N}$ from relation (3.2) because it has characteristic functions H of the operator $\xi_n,\ n\in\mathbb{N}$ corresponding to the eigenvalues $f_n(x,y)$ eigenfunctions are determined by equality (3.3). Theorem 3.1 is proved.

$L_2(\Omega_1\times\Omega_1)$ in hilbert space H we consider the operator as follows:

$$H=H_0-K,(3.7)$$

ISSN 2277-3630 (online), Published by International journal of Social Sciences & Interdisciplinary Research., under Volume: 13 Issue: 04 in April-2024
https://www.gejournal.net/index.php/IJSSIR

Copyright (c) 2024 Author (s). This is an open-access article distributed under the terms of Creative Commons Attribution License (CC BY). To view a copy of this license, visit https://creativecommons.org/licenses/by/4.0/

here

$$H_0 f(x,y) = (1 - cos\pi y) f(x,y),$$
$$K f(x,y) = \int_{\Omega_1}\int_{\Omega_1} e^{a|x-s|} f(s,t) ds dt, \ a < 0. \quad (3.8)$$

Confirmation 3.1. K (3.8) the integral operator is infinite $\eta_n = -\dfrac{2a}{(\pi n + \varepsilon_n)^2 + a^2}, \quad n \in \mathbb{N}$ will have positive eigenvalues, where $0 < \varepsilon_n < \dfrac{\pi}{2}, \lim_{n\to\infty} \varepsilon_n = 0$. K of the integral operator η_n eigenfunctions corresponding to eigenvalues $g_n(x,y) = \varphi_n(x) \cdot \psi_0(y)$ is defined by equality, where $\psi_0(y) \equiv 1$ and

$$\varphi_n(x) = c_n\left(-\frac{\omega_n}{a} cos\omega_n x + sin\omega_n x\right), \ \omega_n = \pi n + \varepsilon_n,$$

$$c_n = \frac{1}{\sqrt{\frac{1}{2}\left(1 + \frac{\omega_n^2}{a^2}\right) + \frac{sin2\omega_n}{4\omega_n}\left(\frac{\omega_n^2}{a^2} - 1\right) - \frac{sin^2\omega_n}{a}}}, \ n \in \mathbb{N}.$$

Proof. K to find the eigenvalue of the operator defined by the following equation K_0 we find the eigenvalues of the operator:

$$K_0\varphi(x) = \int_0^1 e^{a|x-s|}\varphi(s) ds, \ a < 0, \ \varphi(x) \in L_2[0,1] \quad (3.9)$$

for this $K_0\varphi(x) = \eta\varphi(x), \eta > 0$ we look at Eq. From this we create the following equation

$$\varphi(x) = \frac{1}{\eta} e^{ax}\int_0^x e^{-as}\varphi(s) ds + \frac{1}{\eta} e^{-ax}\int_x^1 e^{as}\varphi(s) ds.$$

These are two sides of the equation x by differentiating twice with respect to , we form the following second-order differential equation

$$\varphi''(x) = a^2\varphi(x) + 2\frac{1}{\eta} a\varphi(x). \quad (3.10)$$

(3.10) for the differential equation

$$\varphi'(0) = -a\varphi(0), \ \varphi'(1) = a\varphi(1) \quad (3.11)$$

the boundary condition is appropriate.

We introduce the following definition $\omega^2 = -\left(a^2 + \dfrac{2a}{\eta}\right)$. Let's assume $\omega \neq 0$.

In that case $\varphi(x) = c\left(\dfrac{\omega}{-a} cos\omega x + sin\omega x\right), \ c = const$ function (3.11) is a solution of differential equation (3.10) with boundary conditions, where ω - the solution of the following equation

$$\frac{a^2 - \omega^2}{a\omega} = 2ctg\omega. \quad (3.12)$$

ISSN 2277-3630 (online), Published by International journal of Social Sciences & Interdisciplinary Research., under Volume: 13 Issue: 04 in April-2024
https://www.gejournal.net/index.php/IJSSIR

Copyright (c) 2024 Author (s). This is an open-access article distributed under the terms of Creative Commons Attribution License (CC BY). To view a copy of this license, visit https://creativecommons.org/licenses/by/4.0/

The following $0 < \varepsilon_n < \frac{\pi}{2}, \lim\limits_{n\to\infty} \varepsilon_n = 0$ that satisfies the conditions $\{\varepsilon_n\}_{n\in\Box}$ each of which can be found in sequence $\omega_n = \pi n + \varepsilon_n, \ n \in \Box$ number will be the solution of equation (3.12). On the other hand $-a^2 - \frac{2a}{\eta_n} = (\pi n + \varepsilon_n)^2, n \in \Box$ from equality K_0 operator is positive η_n for eigenvalues $\eta_n = -\frac{2a}{(\pi n + \varepsilon_n)^2 + a^2}, n \in \Box$ we form the equation.

K_0 of the operator η_n orthonormal eigenfunctions corresponding to eigenvalues $\varphi_n(x) = c_n\left(-\frac{\omega_n}{a} cos\omega_n x + sin\omega_n x\right)$ will appear here

$$c_n = \frac{1}{\sqrt{\frac{1}{2}\left(1+\frac{\omega_n^2}{a^2}\right) + \frac{sin2\omega_n}{4\omega_n}\left(\frac{\omega_n^2}{a^2} - 1\right) - \frac{sin^2\omega_n}{a}}}.$$

Ravshanki, K_0 (3.9) is an eigenvalue of the operator K (3.8) is also an eigenvalue of the operator and $g_n(x,y) = \varphi_n(x)\cdot\psi_0(y), n \in \Box$ sequence of functions K is a sequence of orthonormal eigenfunctions of the operator. By Theorem 3.1 H (3.7) it follows that the operator has an infinite number of negative eigenvalues.

Confirmation 3.2. H (3.7) for negative eigenvalues of the operator $\xi_n = 1 - \sqrt{1+\eta_n^2}, \ n \in \Box$ equality is appropriate.

Proof. H we find the negative eigenvalues of the operator. $\xi < 0$ being

$$\Phi(\xi) = \int\limits_{\Omega_1}\int\limits_{\Omega_1} \frac{dxdy}{u(x,y)-\xi} = \int\limits_{\Omega_1} \frac{dy}{u_0(y)-\xi} = \int_0^1 \frac{dy}{1-cos\pi y - \xi} = \frac{1}{\sqrt{-\xi}\sqrt{2-\xi}}$$

we will have a relationship. (3.5) from Eq $(-\xi)(2-\xi) = \eta_n^2, \xi < 0, \ n \in \Box$ we form the equation of this equation $\xi < 0$ as it is $\xi_n = 1 - \sqrt{1+\eta_n^2}, \ n \in \Box$ the solution H operator will have eigenvalues.

4. $\Box \times \Box$ discrete Schrödinger operator on a lattice

Lattice two-particle Hamiltonian $l_2(\Box \times \Box)$ in the Gilbert space we define as follows [15]

$$Q = Q_0 - Q',$$

here is a function wrapper Q_0 The general form of kinetic energy is:

$$(Q_0\phi)(m,n) = \sum_{k,l\in\Box} v_0(m-k, n-l)\phi(k,l),$$

Q' and potential energy

$$(Q'\phi)(m,n) = v_1(m,n)\phi(m,n)$$

defined by Eq. Kinetic energy $v_0(m,n) = u_1(m)\cdot u_2(n)$ is given in the form here

$$u_1(m) = \begin{cases} 2a_1, & \text{agar } m = 0, \\ a_1, & \text{agar } |m| = 1, \\ 0, & \text{agar } m \in \Box \ , \ \{-1;0;1\} \end{cases}, \quad u_2(n) = \begin{cases} 2a_2, & \text{agar } n = 0, \\ a_2, & \text{agar } |n| = 1, \\ 0, & \text{agar } n \in \Box \ , \ \{-1;0;1\} \end{cases}$$

ISSN 2277-3630 (online), Published by International journal of Social Sciences & Interdisciplinary Research., under Volume: 13 Issue: 04 in April-2024
https://www.gejournal.net/index.php/IJSSIR
Copyright (c) 2024 Author (s). This is an open-access article distributed under the terms of Creative Commons Attribution License (CC BY). To view a copy of this license, visit https://creativecommons.org/licenses/by/4.0/

and $a_1, a_2 > 0$. We define the potential function with the following equation

$$v_1(m,n)=\begin{cases}\alpha_{0,0}, & \text{agar } m=n=0,\\ \alpha_{p,0}, & \text{agar } m\in\{\pm p\}, n=0,\ p\in\mathbb{N},\\ \alpha_{0,q}, & \text{agar } m=0, n\in\{\pm q\}, q\in\mathbb{N},\\ \alpha_{p,q}, & \text{agar } m\in\{\pm p\}, n\in\{\pm q\},\ p,q\in\mathbb{N},\end{cases},$$

here $\alpha_{p,q}>0,\ p,q\in\mathbb{N}\cup\{0\},\ \sum_{p,q\in\mathbb{N}\cup\{0\}}\alpha_{p,q}^2<\infty$.

let's say $\mathrm{T}=(-\pi,\pi]$ let it be $\mathrm{F}:l_2(\mathbb{Z}\times\mathbb{Z})\to L_2(\mathrm{T}\times\mathrm{T})$ - is a Fourier substitution, $\mathbb{Z}\times\mathbb{Z}$ on the fence $\phi(m,n)$ function $\mathrm{T}\times\mathrm{T}$ defined in the set $f(x,y)$ reflects to the function, i.e

$$\phi(m,n)\to f(x,y)=\frac{1}{2\pi}\sum_{p,q\in\mathbb{Z}}\phi(p,q)exp(i[(p,x)+(q,y)]),$$

in this Q Hamiltonian $L_2(\mathrm{T}\times\mathrm{T})$ passes to the following operator in space

$$H_2f(x,y)=H_0^{(2)}f(x,y)-K_2f(x,y).\quad(4.1)$$

In this

$$H_0^{(2)}f(x,y)=u_0^{(2)}(x,y)f(x,y),\ \ K_2f(x,y)=\int_{\mathrm{T}}\int_{\mathrm{T}}k_2(x,y;s,t)f(s,t)dsdt$$

and $u_0^{(2)}(x,y)=4a_1a_2(1+cosx)(1+cosy)$, $u_0^{(2)}(\pm\pi,\pm\pi)=0$, $k_2(x,y;s,t)$ - the kernel will have a non-segregated view ie:

$$k_2(x,y;s,t)=\lambda_0\varphi_0(x,y)+\sum_{p=1}^{\infty}\lambda_p\varphi_p^{(1)}(x)\varphi_p^{(1)}(s)+\sum_{p=1}^{\infty}\lambda_p\varphi_p^{(2)}(x)\varphi_p^{(2)}(s)+\sum_{q=1}^{\infty}\lambda_q\varphi_q^{(3)}(y)\varphi_q^{(3)}(t)+$$

$$\sum_{q=1}^{\infty}\lambda_q\varphi_q^{(4)}(y)\varphi_q^{(4)}(t)+\sum_{p,q=1}^{\infty}\lambda_{p,q}\varphi_{p,q}^{(5)}(x,y)\varphi_{p,q}^{(5)}(s,t)+\sum_{p,q=1}^{\infty}\lambda_{p,q}\varphi_{p,q}^{(6)}(x,y)\varphi_{p,q}^{(6)}(s,t)+$$

$$+\sum_{p,q=1}^{\infty}\lambda_{p,q}\varphi_{p,q}^{(7)}(x,y)\varphi_{p,q}^{(7)}(s,t)+\sum_{p,q=1}^{\infty}\lambda_{p,q}\varphi_{p,q}^{(8)}(x,y)\varphi_{p,q}^{(8)}(s,t),$$

here

$$\lambda_0=2\pi\alpha_{0,0},\ \lambda_p=4\pi^2\alpha_{p,0},\ \lambda_q=4\pi^2\alpha_{0,q},\ \lambda_{p,q}=4\pi^2\alpha_{p,q};$$

$$\varphi_0(x,y)=\frac{1}{2\pi},\ \varphi_p^{(1)}(x)=\frac{cospx}{\sqrt{2\pi}},\ \varphi_p^{(2)}(x)=\frac{sinpx}{\sqrt{2\pi}},\ \varphi_q^{(3)}(y)=\frac{cosqy}{\sqrt{2\pi}},\ \varphi_q^{(4)}(y)=\frac{sinqy}{\sqrt{2\pi}},$$

$$\varphi_{p,q}^{(5)}(x,y)=\frac{cospx\cdot cosqy}{\pi},\ \ \varphi_{p,q}^{(6)}(x,y)=\frac{cospx\cdot sinqy}{\pi},$$

$$\varphi_{p,q}^{(7)}(x,y)=\frac{sinpx\cdot cosqy}{\pi},\ \ \varphi_{p,q}^{(8)}(x,y)=\frac{sinpx\cdot sinqy}{\pi}.$$

The following system of functions $L_2(\mathrm{T}\times\mathrm{T})$ it is not difficult to check that it is orthonormal in space

$$\varphi_0,\ \varphi_p^{(1)},\ \varphi_p^{(2)},\ \varphi_q^{(3)},\ \varphi_q^{(4)},\ \varphi_{p,q}^{(5)},\ \varphi_{p,q}^{(6)},\ \varphi_{p,q}^{(7)},\ \varphi_{p,q}^{(8)},\ \ p,q\in\mathbb{N}\quad(4.2)$$

Besides

$$K_2\varphi_0(x,y)=\lambda_0\varphi_0(x,y),\ K_2h_p^{(1)}(x,y)=\lambda_ph_p^{(1)}(x,y),\ \ K_2h_p^{(2)}(x,y)=\lambda_ph_p^{(2)}(x,y)$$

ISSN 2277-3630 (online), Published by International journal of Social Sciences & Interdisciplinary Research., under Volume: 13 Issue: 04 in April-2024
https://www.gejournal.net/index.php/IJSSIR

Copyright (c) 2024 Author (s). This is an open-access article distributed under the terms of Creative Commons Attribution License (CC BY). To view a copy of this license, visit https://creativecommons.org/licenses/by/4.0/

$$K_2h_q^{(3)}(x,y)=\lambda_q h_q^{(3)}(x,y),\ \ K_2h_q^{(4)}(x,y)=\lambda_q h_q^{(4)}(x,y)$$
$$K_2\varphi_{p,q}^{(5)}(x,y)=\lambda_{p,q}\varphi_{p,q}^{(5)}(x,y),\ \ K_2\varphi_{p,q}^{(6)}(x,y)=\lambda_{p,q}\varphi_{p,q}^{(6)}(x,y)$$
$$K_2\varphi_{p,q}^{(7)}(x,y)=\lambda_{p,q}\varphi_{p,q}^{(7)}(x,y),\ \ K_2\varphi_{p,q}^{(8)}(x,y)=\lambda_{p,q}\varphi_{p,q}^{(8)}(x,y)$$

equalities are appropriate, here $h_p^{(1)}(x,y)=\varphi_p^{(1)}(x)\cdot\psi_0(y),\ \ h_p^{(2)}(x,y)=\varphi_p^{(2)}(x)\cdot\psi_0(y),$
$h_q^{(3)}(x,y)=\varphi_q^{(3)}(y)\cdot\psi_0(x),\ \ h_q^{(4)}(x,y)=\varphi_q^{(4)}(y)\cdot\psi_0(x),\ \ \psi_0(y)=\psi_0(x)\equiv 1.$
Thus, every function in the system (4.2). K_2 characteristic function of the operator,

$$\lambda_0=2\pi\alpha_{0,0},\ \lambda_p=4\pi^2\alpha_{p,0},\ \lambda_q=4\pi^2\alpha_{0,q},\ \lambda_{p,q}=4\pi^2\alpha_{p,q}\ \ (p,q\in\square)$$

while the numbers K_2 will be the eigenvalues of the operator.

Theorem 4.1. H_2 the discrete Schrödinger operator has an infinite number of negative eigenvalues.

Proof. H_2 The operator (4.1) is the operator in the Friedrichs model. let's say β - hip $\beta\geq 8a_1a_2$ be an arbitrary positive number satisfying the inequality.
$L_2(\mathrm{T}\times\mathrm{T})$ in the Friedrichs model in space H_1 we define the operator as follows

$$H_1=H_0^{(1)}-K_1,\quad(4.3)$$

in this

$$H_0^{(1)}f(x,y)=\beta(1+cosy)f(x,y),\quad K_1f(x,y)=\int_{\mathrm{T}}\int_{\mathrm{T}}k_1(x;s)f(s,t)dsdt,$$

here

$$k_1(x;s)=\lambda_0\varphi_0(x)+\sum_{p=1}^{\infty}\lambda_p\varphi_p^{(1)}(x)\varphi_p^{(1)}(s)+\sum_{p=1}^{\infty}\lambda_p\varphi_p^{(2)}(x)\varphi_p^{(2)}(s).$$

Ravshanki, $\lim\limits_{\xi\to 0-0}\int_{\mathrm{T}}\frac{dy}{1+cosy-\xi}=+\infty$ equality is appropriate. Then by Theorem 3.1 H_1 (4.3) operator has an infinite number of negative eigenvalues. On the other hand, $H_0^{(1)}\geq H_0^{(2)}$ and $K_2\geq K_1$ since the inequalities hold, the proof of Theorem 4.1 follows from Theorem 2.1.

List of references

1. Faddeev L.D. O model of Friedrichs and the theory of spatial noninterruptive spectrum. V. kn.: Trudy MI AN SSSR, T.73, M.: Nauka, 1964, S. 292-313.
2. Lakaev S.N., Minlos R.A. O svyazannyx sostoyaniyax clusternogo operatora TMF. 1979. T.39. #1. S. 83-93.
3. Minlos R.A., Sinai Ya.G. Issledovanie spektrov stochasticheskikh operatorov, voznikayushchix v reshetchatyx modeliax gasa. TMF. 1970. T.2. #2. S. 230-243.
4. Reid M., Simon B. Methods of modern mathematical physics, Volume 4: Analysis of operators. -- M.: Mir, 1982.
5. Eshkabilov Yu. X. Ob odnom operatora v modeli Fridrikhsa, UzMJ, 1999, No. 3, str. 85-93.
6. Eshkabilov Yu. X. O beskonechnosti discretnogo spectra operatorov v model Friedrichsa. Mat. trudy. 2011. T.14. #1. S. 195-211.

ISSN 2277-3630 (online), Published by International journal of Social Sciences & Interdisciplinary Research., under Volume: 13 Issue: 04 in April-2024
https://www.gejournal.net/index.php/IJSSIR
Copyright (c) 2024 Author (s). This is an open-access article distributed under the terms of Creative Commons Attribution License (CC BY). To view a copy of this license, visit https://creativecommons.org/licenses/by/4.0/

7. Eshkabilov Yu. X. O beskonechnosti chisla otritsatelnyx sobstvennyx znacheniy model Friedrichsa. Nanosystem: physics, chemistry, mathematics, 2012, 3(6), S. 16-24.

8. Eshkabilov Yu. Kh., Kulturaev D. J. On the infinity of the discrete spectrum of operators and the multidimensional model of Friedrichs. News of UzMU, 2014, No. 1.

9. Abdullaev J. I. Sobstvennye znacheniya dvukhchastichnogo operatora Schrödinger na dvumernoy reshetke. Uzbek. matem. journal., 2005, No. 1, S. 3-11.

10. Lusternik L. A., Sobolev V. I. Elementy functional analysis. Moscow: Nauka. 1965.

11. Imamkulov S. A., Lakaev S. N. Discrete spectrum one-dimensional model Friedrichsha. Dokl. AN UzSSSR. 1988. #7. S. 9-11.

12. Lakaev S.N. O discrete spectrum obobshchennoy model Friedrichs. Doc. AN UzSSR. 1979. No. 4. S. 9-10.

13. Lakaev S. N. Some spectral properties of the general model of Friedrichsha. Tr.seminar N.G.Petrovsky. 1986. No. 11. S. 210-238.

14. Minlos RA, Abdullaev JI, Lakaev SN On the Spectral Properties of the Matrix-valued Friedrichs Model Many-Particle Hamiltonians:

Spectrum and Scattering. Editor: //Advances in Soviet Mathematics, 1991, 5.

15. Zhukov Yu. V. Theorem Iorio-O'Carroll for N-partial lattice Hamiltonian // TMF, 107:1, 1996, 75-85.

16. Reid M., Simon B. Methods of modern mathematical physics - T.1, Functional analysis. - M.; Mir, 1977.

17. Trichomi F. Dj. Integral equations. IL, M., 1960.

ISSN 2277-3630 (online), Published by International journal of Social Sciences & Interdisciplinary Research., under Volume: 13 Issue: 04 in April-2024
https://www.gejournal.net/index.php/IJSSIR

Copyright (c) 2024 Author (s). This is an open-access article distributed under the terms of Creative Commons Attribution License (CC BY). To view a copy of this license, visit https://creativecommons.org/licenses/by/4.0/