MATHEMATICAL MODELING OF THE SYSTEM'S MOTION, THE STABILITY OF WHICH IS BEING STUDIED

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Annotation: Mathematical modeling of the motion of dynamic or mechanical systems plays an important role in the stability of the system's motion. The structure of its mathematical model is especially important when studying the stability of the motion of complex systems. Therefore, when creating a mathematical model of the motion of such systems, it is important to consider the study of the stability of the system's motion in the future.

Keywords: mathematical modeling, stability, asymptotic stability, large-scale systems, mechanical systems.

Life consists of motion, and when studying motion, it must be mathematically modeled. In most cases, the optimal option is sought when mathematically modeling the motion of a material point, solid, mechanical or dynamic system. That is, the appearance of the mathematical model should be simple, convenient, and the error in the solution should not be large. But one of the main characteristics of any movement is the question of its stability.

The main part.

There are such mechanical or dynamic systems that are so important in human life that they are very complex, and if mathematical modeling itself is complex, then studying the stability of this movement becomes even more difficult. As an example, one can cite the model of neural networks. Neural networks are a system of neurons in the human brain, the movement of information (signs) in it is a very complex movement. Given that the neural network system is multiparametric and fully controls human activity, it is necessary to fully understand the movement of this system. This means that it is necessary to study the motion of a very complex system.

The need to create and implement a model for modeling The purpose of modeling is determined before creating a model, the results of modeling are analyzed after application, and the problem of stability, which is one of its main properties, is investigated.

Determine the sufficient conditions for the asymptotic stability of a system whose stiffness and damping are nonlinear and clearly dependent on time.

The excited motion of various systems is very often described by one second-order differential equation

$$\ddot{x} + \beta(t, x, \dot{x})\dot{x} + \alpha(t, x, \dot{x})x = 0 \qquad (1)$$

where the positive real functions and real variables are defined in the region $t \ge t_0, \quad x^2 + \dot{x}^2 \le \mu$ (2)

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(t_0, μ - some positive constants).

Function $\beta(t,x,\dot{x})$ can be interpreted as a nonlinear, time-dependent, generalized damping coefficient, and the function $\alpha(t,x,\dot{x})$ - as a nonlinear, obviously time-dependent generalized rigidity of the system. With any, but constant and positive coefficients α and β leisurely movement $x=0,\ \dot{x}=0$ equations (1) are asymptotically stable. If these coefficients, remaining positive, change, there are modes of their change under which the motion becomes unstable. When the law of change of coefficients α and β it is known that one can apply a particular method and investigate the stability of motion. However, there are cases in applications where the nature of functions α and β They are not defined and only the limits of their change in the region (2) are known
$a_1 \le \alpha(t,x,\dot{x}) \le a_2, \quad b_1 \le \beta(t,x,\dot{x}) \le b_2,$ (3)

where a_1, a_2, b_1, b_2 - given positive numbers (the case $a_1 = 0$ or $b_1 = 0$ we consider excluded)

Therefore, it is interesting to determine the conditions for a_1, a_2, b_1, b_2, which are performed without disturbed movement $x=0,\ \dot{x}=0$ will be asymptotically stable under any law of change of the function α and β within specified limits. (We assume that the functions α and β We assume that for all $t, x, \dot{x}$ from the region (2), they satisfy the conditions for the existence and uniqueness of the solution of the equation (1).

First of all, let's note that the condition $a_1 > 0,\ b_1 > 0$ necessary. Indeed, if, for example, $b_1 \le 0$, then, using arbitrariness α and β, suppose $\alpha = const,\ \beta = b_1 \le 0$. With these values α and β unsteady motion at $b_1 < 0$ and stable, but not asymptotically $b = 0$ and $\alpha = cont > 0$.

Let's move on to the task at hand. By substituting

$$x = x_1, \quad \dot{x} = x_2$$

let's replace equation (1) with an equivalent system
$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -\alpha(t,x,\dot{x})x_1 - \beta(t,x,\dot{x})x_2$$

or in matrix form

$$\dot{y} = A(t,y)y, \tag{4}$$

Where $y = (x_1, x_2)^T$, $A(t,y) = \begin{pmatrix} 0 & 1 \\ -\alpha(t,y) & -\beta(t,y) \end{pmatrix}$

Let's introduce strictly positive functions $\varphi_1(t,y)$ and $\varphi_2(t,y)$ rigidly ε_1 and ε_2 respectively, by equality

$$\varphi_1(t,y) = m_1 - \alpha(t,y), \quad \varphi_2(t,y) = m_2 - \beta(t,y) \tag{5}$$

It is not difficult to check that when the inequalities (3) are satisfied, the function $\varphi_1(t,y)$ and $\varphi_2(t,y)$ satisfy ratings

$$m_1 - a_2 \le \varphi_1(t,y) \le m_1 - a_1, \quad m_2 - b_2 \le \varphi_2(t,y) \le m_2 - b_1 \tag{6}$$

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where does it come from $m_1 = a_2 + \varepsilon_1$, $m_2 = b_2 + \varepsilon_2$. Therefore, inequalities (6) can be rewritten as follows

$$\varepsilon_1 \le \varphi_1(t,y) \le a_2 - a_1 + \varepsilon_1, \quad \varepsilon_2 \le \varphi_2(t,y) \le b_2 - b_1 + \varepsilon_1$$

Using (5), the system (4) can be represented as follows:

$$\dot{y} = A_0 y + \varphi_1(t,y) A_1 y + \varphi_2(t,y) A_2 y, \tag{7}$$

Where $A_0 = \begin{pmatrix} 0 & 1 \\ -m_1 & -m_2 \end{pmatrix}$, $A_1 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$, $A_2 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$,

For system (7) we construct Lyapunov functions in the following form:

$$V(y) = y^T P y \tag{8}$$

where $P = \begin{pmatrix} \dfrac{m_1(m_1+1)}{m_2} & 1 \\ 1 & \dfrac{m_1+1}{m_2} \end{pmatrix}$

is positively determined.

With this matrix selection P, for the derivative of function (8) by the system (7) we obtain

$$\dot{V}(y) = \dot{y}^T P y + y^T P \dot{y} = y^T (G_0 + \varphi_1(t,y) G_1 + \varphi_2(t,y) G_2) y,$$

wrehe $G_0 = \begin{pmatrix} -2m_1 & 0 \\ 0 & -2m_1 \end{pmatrix}$, $G_1 = \begin{pmatrix} 2 & \dfrac{m_1+1}{m_2} \\ \dfrac{m_1+1}{m_2} & 0 \end{pmatrix}$, $G_2 = \begin{pmatrix} 0 & 1 \\ 1 & 2\dfrac{m_1+1}{m_2} \end{pmatrix}$

and

$$\dot{V}(y) \le (\lambda_M(G_0) + (a_2 - a_1 + \varepsilon_1)\lambda_M(G_1) + (b_2 - b_1 + \varepsilon_2)\lambda_M(G_2)) \|y\|^T, \tag{9}$$

where $\lambda_M(G_0) = -2m_1$, $\lambda_M(G_1) = 1 + \sqrt{1 + \left(\dfrac{m_1+1}{m_2}\right)^2}$, $\lambda_M(G_2) = \dfrac{m_1+1}{m_2} + \sqrt{1 + \left(\dfrac{m_1+1}{m_2}\right)^2}$.

From the estimate (9) it follows that the function is negative if

$$\lambda_M(G_0) + (a_2 - a_1 + \varepsilon_1)\lambda_M(G_1) + (b_2 - b_1 + \varepsilon_2)\lambda_M(G_2) < 0, \tag{10}$$

It follows from this that, based on Lyapunov's asymptotic stability theorem, the smooth motion of the system (10) or (1) is asymptotically stable if the inequalities are satisfied (10).

Let it be $\dfrac{m_1+1}{m_2} = k$, Then the inequality (10) has the form:

$$-2m_1 + (m_1 - a_1)(1 + \sqrt{1+k^2}) + (m_2 - b_1)(k - \sqrt{1+k^2}) < 0,$$

or

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$$(m_1 + m_2 - a_1 - b_1)\sqrt{1+k^2} < a_1 + b_1 k - 1.$$

This inequality makes sense if $a_1 + b_1 k - 1 > 0$.

From these inequalities, $\varepsilon_1 \to 0, \ \ \varepsilon_2 \to 0$ и $k = \dfrac{a_2 + 1 + \varepsilon_1}{b_2 + \varepsilon_2}$, we get

$$k = \frac{m_1 + 1}{m_2} = \frac{a_2 + 1}{b_2}, \quad \sqrt{1+k^2} = \sqrt{1 + \frac{(b_2+1)^2}{b_2^2}} = \frac{\sqrt{b_2^2 + (a_2+1)^2}}{b_2}$$

$$a_2 - a_1 + b_2 - b_1 < \frac{1}{\sqrt{b_2^2 + (a_2+1)^2}}\left[(a_2+1)b_1 + (a_1-1)b_2\right] \qquad (11)$$

$$(a_2+1)b_1 + (a_1-1)b_2 > 0 \qquad (12)$$

Thus, if the boundaries a_1, a_2, b_1, b_2 functions $\alpha(t, x, \dot{x})$ and $\beta(t, x, \dot{x})$ satisfying the conditions (11) and (12), then the unrestricted motion $x = 0$, $\dot{x} = 0$ asymptotically stable.

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