ON ONE BOUNDARY PROBLEM FOR A PARABOLIC-HYPERBOLIC EQUATION OF THE THIRD ORDER, WHEN THE CHARACTERISTIC OF THE FIRST ORDER OPERATOR IS PARALLEL TO THE YORDINATE AXIS

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Annotation: *In the present paper, one boundary value problem third is posed and studied for a-order parabolic-hyperbolic equation in a quadrangular domain with two lines of type change.*

I. Introduction.

Many problems in physics, engineering, mechanics and other areas require the study of equations of mixed, composite and mixed-composite types.

Fundamental research into mixed equations of the second order of elliptic-hyperbolic type began to be studied by the Italian mathematician Tricomi in the 20s of the last century [1].

After that, many different problems for equations of these types began to be investigated. A review of theoretical and applied research is given in the works and books of A.V. Bitsadze [2, 3], L. Bers [4], M.M. Smirnov [5], and also in the books of M.S., T.D. Dzhuraeva [7].

Research into equations of elliptic-parabolic, parabolic-hyperbolic types began in the 1950s and 1960s. In 1959, I.M. Gel'fand [8] pointed out the need for joint consideration of equations in one part of the domain of parabolic, and the other part - of hyperbolic types. He gives an example related to the movement of gas in a channel surrounded by a porous medium: in the channel, the movement of gas is described by the wave equation, outside it - by the diffusion equation.

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Then, in the 1970s and 1980s, they began to study various problems for equations of the third and higher orders of the parabolic-hyperbolic type. Such problems were studied mainly by T. D. Dzhuraev and his students (for example, see [9]-[13]).

II. Formulation of the problem

Let us be given a quadrangular area G on the plane xOy with vertices at the points $D(-1;0)$, $C(2,0)$, $B_0(1;1)$, $A_0(0,1)$ and let the area G have the form $G = G_1 \cup G_2 \cup G_3 \cup J_1 \cup J_2$, where G_1 – the rectangle with vertices at the points $A(0;0)$, $B(1;0)$, $B_0(1,1)$, $A_0(0,1)$; G_2 – triangle with vertices at points $A(0;0)$, $D(-1;0)$, $A_0(0,1)$; G_3 – triangle with vertices at points $B(1;0)$, $B_0(1,1)$, $C(2,0)$; J_1 – open segment with vertices at points $A(0;0)$, $A_0(0,1)$; J_2 – open segment with vertices at points $B(1;0)$, $B_0(1,1)$.

In the area G, consider the equation

$$\left(b\frac{\partial}{\partial y}+c\right)(Lu)=0 \qquad (1)$$

where $b,c \in R$, $Lu \equiv \begin{cases} u_{1xx}-u_{1y}, & (x,y)\in G_1, \\ u_{ixx}-u_{iyy}, & (x,y)\in G_i \ (i=2,3,4). \end{cases}$

For equation (1), the following problem is posed:

Problem 1. Find a function $u(x,y)$ that 1) is continuous in $\overline{G}$ and $G \setminus J_1 \setminus J_2 \setminus J_3$ has continuous derivatives involved in equation (1), and u_x, u_y and u_{yy} are continuous up to a part of the boundary of the region G specified in the boundary conditions; 2) satisfies equation (1) in the region $G \setminus J_1 \setminus J_2 \setminus J_3$; 3) satisfies the following boundary conditions:

$$u(x,0)=f_1(x), \quad 0\le x\le 1, \qquad (2)$$

$$u_y(x,0)=f_2(x), \quad 0\le x\le 1, \qquad (3)$$

$$u(x,0)=f_3(x), \quad -1\le x\le 0, \qquad (4)$$

$$u_y(x,0)=f_4(x), \quad -1\le x\le 0, \qquad (5)$$

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$$u_{yy}(x,0)=f_5(x),\ -1<x<0, \quad (6)$$

$$u(x,0)=f_6(x),\ 1\le x\le 2, \quad (7)$$

$$u_y(x,0)=f_7(x),\ 1\le x\le 2, \quad (8)$$

$$u_{yy}(x,0)=f_8(x),\ 1<x<2, \quad (9)$$

and 4) satisfies the following continuous gluing conditions:

$$u(+0,y)=u(-0,y)=\tau_1(y),\ 0\le y\le 1, \quad (10)$$

$$u_x(+0,y)=u_x(-0,y)=\nu_1(y),\ 0\le y\le 1, \quad (11)$$

$$u(1+0,y)=u(1-0,y)=\tau_2(y),\ 0\le y\le 1, \quad (12)$$

$$u_x(1+0,y)=u_x(1-0,y)=\nu_2(y),\ 0\le y\le 1. \quad (13)$$

Here $f_i\ (i=\overline{1,8})$, are given sufficiently smooth functions, and $\tau_1,\ \nu_1,\ \tau_2,\ \nu_2$ are unknown yet sufficiently smooth functions to be determined.

Theorem. If $f_1\in C^3[0,1]$, $f_3\in C^3[-1,0]$, $f_6\in C^3[1,2]$, $f_2\in C^2[0,1]$, $f_4\in C^2[-1,0]$, $f_7\in C^2[1,2]$, $f_5\in C^1[-1,0]$, $f_8\in C^1[1,2]$, and the following matching conditions are satisfied: $\tau_1(0)=f_1(0)=f_3(0)$, $\tau_2(0)=f_1(1)=f_6(1)$, $\nu_1(0)=f_2(0)=f_4(0)$, $\nu_2(0)=f_2(1)=f_7(1)$, then Problem 1 admits a unique solution .

To prove this theorem, we rewrite equation (1) in the form

$$u_{1xx}-u_{1y}=\omega_1(x)e^{-\frac{c}{b}y},\ (x,y)\in G_1, \quad (14)$$

$$u_{ixx}-u_{iyy}=\omega_i(x)e^{-\frac{c}{b}y},\ (x,y)\in G_i\ (i=2,\,3), \quad (15)$$

where $\omega_i(x)\ (i=1,2,3)$ are so far unknown sufficiently smooth functions.

First, we study Problem 1 in the domain G_2. Passing in the equation (15) ($i=2$) to the limit at $y\to 0$, due to (4) and (6) we find

$$\omega_2(x)=f_3''(x)-f_5(x),\ -1\le x\le 0.$$

Further, by the continuation method after long calculations and transformations, we arrive at the relation between the functions $\tau_1(y)$ and $\nu_1(y)$:

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$$\nu_1(y) = \tau_1'(y) + \beta_1(y), \; 0 \le y \le 1 \qquad (16)$$

where $\beta_1(y)-$ known function.

Similarly, passing to the region G_3, at $y \to 0$ from Eq. (15) ($i = 3$) we find function $\omega_3(x)$.

Using the continuation method, as in the area G_2, we obtain the relation between the functions $\tau_2(y)$ and $\nu_2(y)$:

$$\nu_2(y) = -\tau_2'(y) + \beta_2(y), \; 0 \le y \le 1 \qquad (17)$$

where $\beta_2(y)-$ known function .

Next, move on to the area G_1. Assuming in equation (14) $y \to 0$, due to (2) and (3), we find the function $\omega_1(x)$.

Now we write down the solution of equation (14) that satisfies conditions (2), (10), (12) (see [9]):

$$u_1(x,y) = \int_0^y \tau_1(\eta) G_\xi(x,y;0,\eta)d\eta - \int_0^y \tau_2(\eta) G_\xi(x,y;1,\eta)d\eta +$$

$$+\int_0^1 f_1(\xi) G(x,y;\xi,0)d\xi - \int_0^y e^{-\frac{c}{b}\eta} d\eta \int_0^1 \omega_1(\xi) G(x,y;\xi,\eta)d\xi .$$

Differentiating this solution with respect to x, we get:

$$u_{1x}(x,y) = -\int_0^y \tau_1'(\eta) N(x,y;0,\eta)d\eta + \int_0^y \tau_2'(\eta) N(x,y;1,\eta)d\eta +$$

$$+\int_0^1 \tau_1'(\xi) N(x,y;\xi,0)d\xi + \int_0^y e^{-\frac{c}{b}\eta} d\eta \int_0^1 \omega_1(\xi) N_\xi(x,y;\xi,\eta)d\xi , \text{ (eighteen)}$$

where

$$\left.\begin{matrix} G(x,y;\xi,\eta) \\ N(x,y;\xi,\eta) \end{matrix}\right\} = \frac{1}{2\sqrt{\pi(y-\eta)}} \sum_{n=-\infty}^{+\infty} \left\{ \exp\left[-\frac{(x-\xi-2n)^2}{4(y-\eta)}\right] \mp \exp\left[-\frac{(x+\xi-2n)^2}{4(y-\eta)}\right] \right\} -$$

Green's functions of the first and second boundary value problems for Eq. (14).

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Assuming in (18) $x=0$and $x=1$we obtain two relations between the unknown functions $\tau_1(y)$, $\nu_1(y)$, $\tau_2(y)$ and $\nu_2(y)$.

Eliminating from these two relations and from (16), (17) the functions $\nu_1(y)$and $\nu_2(y)$, we arrive at a system of Volterra integral equations of the second kind with respect to $\tau_1'(y)$and $\tau_2'(y)$:

$$\tau_1'(y)+\int_0^y K_1(y,\eta)\tau_1'(\eta)d\eta+\int_0^y K_2(y,\eta)\tau_2'(\eta)d\eta=g_1(y), \qquad (19)$$

$$\tau_2'(y)+\int_0^y K_3(y,\eta)\tau_2'(\eta)d\eta+\int_0^y K_4(y,\eta)\tau_1'(\eta)d\eta=g_2(y), \qquad (20)$$

where $K_1(y,\eta)$, $K_2(y,\eta)$. $K_3(y,\eta)$. $K_4(y,\eta)$. $g_1(y)$. $g_2(y)$– known functions, $K_1(y,\eta)$and $K_3(y,\eta)$ have a weak singularity (with degree $1/2$), while the functions $K_2(y,\eta)$, $K_4(y,\eta)$, $g_1(y)$and $g_2(y)$– are continuous functions.

Solving the system (19), (20), we find the functions $\tau_1'(y)$and $\tau_2'(y)$, and thus the functions $\nu_1(y)$. $\nu_2(y)$. $u_2(x,y)$, $u_3(x,y)$ and $u_1(x,y)$.

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