ON A BOUNDARY VALUE PROBLEM FOR A THIRD-ORDER PARABOLIC-HYPERBOLIC EQUATION IN A PENTAGONAL DOMAIN WITH THREE LINES OF TYPE CHANGE, WHOSE HYPERBOLIC PARTS ARE TRIANGLES

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Annotation: *In the present paper, we pose and third study one boundary value problem for a-order parabolic-hyperbolic equation in a pentagonal domain with three lines of type change, the hyperbolic parts of which are triangles.*

I. Introduction

Fundamental research into mixed equations of the second order of elliptic-hyperbolic type began to be studied by the Italian mathematician Tricomi in the 20s of the last century [1].

After this, many different problems for equations of these types began to be investigated. A review of theoretical and applied research is given in the works and books of A.V. Bitsadze [2, 3], L. Bers [4], M.M. Smirnov [5], and also in the books of M.S. Salakhitdinov [6], T.D. Dzhuraeva [7].

Research into equations of elliptic-parabolic, parabolic-hyperbolic types began in the 1950s and 1960s. Then, in the 1970s and 1980s, they began to study various problems for equations of the third and higher orders of the parabolic-hyperbolic type. Such problems were studied mainly by T.D. Dzhuraev and his students (for example, see [8]-[12]).

II. Formulation of the problem

In the present work, one boundary value problem is posed and for the equation third-order parabolic-hyperbolic type of the form

$$\left(b\frac{\partial}{\partial y}+c\right)(Lu)=0 \qquad (1)$$

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in the area of G the plane xOy, where $G = G_1 \cup G_2 \cup G_3 \cup G_4 \cup J_1 \cup J_2 \cup J_3$, $b, c \in R$,

$$Lu = \begin{cases} u_{1xx} - u_{1y}, & (x,y) \in G_1, \\ u_{ixx} - u_{iyy}, & (x,y) \in G_i \quad (i = 2,3,4), \end{cases} \qquad u(x,y) = u_i(x,y),\ (x,y) \in G_i \quad (i = \overline{1,4}),$$

G_1 – rectangle with vertices at points $A(0;0)$, $B(1;0)$, $B_0(1,1)$, $A_0(0,1)$; G_2 – triangle with vertices at points $E(2,0)$. $C(1/2,-3/2)$. $D(-1,0)$; G_3 – triangle with vertices at points A. D. A_0; G_4 – triangle with vertices at points E. B. B_0; J_1 – open segment with vertices at points E, D; J_2 – open segment with vertices at points A, A_0; J_2 – open segment with vertices at points B, B_0.

Equation (1) is a special case of the equation

$$\left(a\frac{\partial}{\partial x} + b\frac{\partial}{\partial y} + c \right)(Lu) = 0$$

at $a = 0$. For equation (1), the following problem is posed:

Task M_{0bc}**.** It is required to find a function $u(x,y)$ that is 1) continuous in $\overline{G}$ and $G \setminus J_1 \setminus J_2 \setminus J_3$ has in the domain continuous derivatives involved in equation (1), u_x and u_y are continuous up to a part of the boundary of the region G, indicated in the boundary conditions ; 2) satisfies equation (1) in the region $G \setminus J_1 \setminus J_2 \setminus J_3$; 3) satisfies the boundary conditions

$$u|_{EC} = \psi_1(x),\ 1/2 \le x \le 2; \ (2) \qquad u|_{DP} = \psi_2(x),\ -1 \le x \le -1/2; \ (3)$$

$$u|_{QC} = \psi_3(x),\ 0 \le x \le 1/2; \ (4) \qquad \left.\frac{\partial u}{\partial n}\right|_{EC} = \psi_4(x),\ 1/2 \le x \le 2; \qquad (5)$$

$$\left.\frac{\partial u}{\partial n}\right|_{DC} = \psi_5(x),\ -1 \le x \le 1/2; \qquad (6)$$

and 4) satisfies the following bonding conditions:

$$u(x,+0) = u(x,-0) = T(x),\ -1 \le x \le 2; \ (7) \qquad u_y(x,+0) = u_y(x,-0) = N(x),\ -1 \le x \le 2; \ (8)$$

$$u_{yy}(x,+0) = u_{yy}(x,-0) = M(x),\ -1 < x < 2; \ (9) \qquad u(+0,y) = u(-0,y) = \tau_4(y),\ 0 \le y \le 1; \ (10)$$

$$u_x(+0,y) = u_x(-0,y) = \nu_4(y),\ 0 \le y \le 1; \ (11) \qquad u(1-0,y) = u(1+0,y) = \tau_5(y),\ 0 \le y \le 1; \ (12)$$

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$u_x(1-0,y)=u_x(1+0,y)=\nu_5(y),\ 0\le y\le 1$. (13)

Here $\psi_i\ \left(i=\overline{1,5}\right)$ – given functions and , in addition, the notation

$$T(x)=\begin{cases}\tau_1(x), \text{ если } -1\le x\le 0,\\ \tau_2(x), \text{ если } 0\le x\le 1,\\ \tau_3(x), \text{ если } 1\le x\le 2;\end{cases}\quad N(x)=\begin{cases}\nu_1(x), \text{ если } -1\le x\le 0,\\ \nu_2(x), \text{ если } 0\le x\le 1,\\ \nu_3(x), \text{ если } 1\le x\le 2;\end{cases}\quad M(x)=\begin{cases}\mu_1(x), \text{ если } -1\le x\le 0,\\ \mu_2(x), \text{ если } 0\le x\le 1,\\ \mu_3(x), \text{ если } 1\le x\le 2,\end{cases}$$

$\tau_i,\ \nu_i\ (i=\overline{1,5}),\ \mu_j\ \left(j=\overline{1,3}\right)$ – are still unknown sufficiently smooth functions and , n – the inner normal to the line $x+y=-1$ or $x-y=2$, and the points P and Q have coordinates $P(-1/2,-1/2)$, $Q(0,-1)$.

III. Problem research.

Here we give the idea of proving the following theorem.

Theorem. If $\psi_1\in C^3[1/2,2]$, $\psi_2\in C^3[-1,-1/2]$, $\psi_3\in C^3[0,1/2]$, $\psi_4\in C^2[1/2,2]$, $\psi_5\in C^2[-1,1/2]$, and the matching conditions $\psi_1(1/2)=\psi_3(1/2)$, $\tau_2(-1)=\psi_2(-1)$, $\tau_2(0)=\tau_1(0)$, $\tau_2'(0)=\tau_1'(0)$, $\tau_3(1)=\tau_1(1)$, , are satisfied $\psi_4'(1/2)=-\psi_5'(1/2)$, then the problem M_{0bc} admits a unique solution.

Proof. We will prove the theorem by the method of constructing a solution. To do this, we rewrite equation (1) in the form

$$u_{1xx}-u_{1y}=\omega_1(x)e^{-\frac{c}{b}y},\ (x,y)\in G_1, \qquad (14)$$

$$u_{ixx}-u_{iyy}=\omega_i(x)e^{-\frac{c}{b}y},\ (x,y)\in G_i\quad (i=2,3,4), \qquad (15)$$

where $\omega_i(x),\ i=\overline{1,4}$ – unknown yet sufficiently smooth functions to be determined .

Let us first consider the problem in the domain G_2. The solution of equation (15) with $i=2$, satisfying conditions (7), (8) can be represented as

$$u_2(x,y)=\frac{1}{2}\left[T(x+y)+T(x-y)\right]+\frac{1}{2}\int\limits_{x-y}^{x+y}N(t)\,dt-\frac{1}{2}\int\limits_0^y e^{-\frac{c}{b}\eta}d\eta\int\limits_{x-y+\eta}^{x+y-\eta}\omega_2(\xi)\,d\xi. \qquad (16)$$

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Substituting (16) into conditions (5) and (6) after some calculations, we find

$$\omega_2(x) = -\sqrt{2}\psi_4'(x)e^{\frac{c}{b}(x-2)}, \ 1/2 \le x \le 2,$$

$$\omega_2(x) = \sqrt{2}\psi_5'(x)e^{-\frac{c}{b}(1+x)}, \ -1 \le x \le 1/2.$$

From these equalities it follows

$$\psi_4'(1/2) = -\psi_5'(1/2).$$

Further, substituting (16) into condition (2) after some calculations and simplifications, we obtain the first relation between the unknown traces of the solution on the type change line J_1:

$$T'(x) + N(x) = \alpha_1(x), \ -1 \le x \le 2, \ (17)$$

where $\alpha_1(x)$ – is a known function.

In the interval $0 \le x \le 1$, relation (17) has the form

$$\tau_1'(x) + \nu_1(x) = \alpha_1(x), \ 0 \le x \le 1. \ (18)$$

in between $-1 \le x \le 0$ -

$$\tau_2'(x) + \nu_2(x) = \alpha_1(x), \ -1 \le x \le 0, \ (19)$$

and in between $1 \le x \le 2$ -

$$\tau_3'(x) + \nu_3(x) = \alpha_1(x), \ 1 \le x \le 2. \ (20)$$

Now substituting (16) into condition (3), we have

$$\tau_2'(x) - \nu_2(x) = \delta_1(x), \ -1 \le x \le 0, \ (21)$$

where $\delta_1(x)$ – is a known function.

Solving system (19), (21), we find the functions $\tau_2'(x)$, $\nu_2(x)$. Integrating $\tau_2'(x)$ from -1 to x, we determine the function $\tau_2(x)$.

Further, substituting (16) into condition (4), we have

$$\tau_3'(x) - \nu_3(x) = \delta_2(x), \ 1 \le x \le 2, \ (22)$$

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where $\delta_2(x)$ – is a known function.

Solving system (20), (22), we find the functions $\tau_3'(x)$, $\nu_3(x)$. Integrating $\tau_3'(x)$ from 2 to x, we determine the function $\tau_3(x)$.

Finally, we rewrite equation (1) in the form

$$bu_{1xxy} + cu_{1xx} - bu_{1yy} - cu_{1y} = 0.$$

Passing in this equation and in equation (15) ($i = 2$) to the limit at $y \to 0$, we obtain the second and third relations between the unknown functions $\tau_1(x)$, $\nu_1(x)$ and $\mu_1(x)$ on the type change line J_1. Eliminating the functions and $\mu_1(x)$ from these two equations and from Eq. (18) $\nu_1(x)$, we arrive at an ordinary differential equation for $\tau_1(x)$. Solving the resulting equation under known three conditions, we find the function $\tau_1(x)$.

Thus, we have found the function $u_2(x,y)$ in the domain G_2 completely.

In the domains G_3 and G_4 by the continuation method, we obtain two relations between the unknown functions $\tau_4(y)$, $\nu_4(y)$ and $\tau_5(y)$, $\nu_5(y)$.

Then, in the domain G_1, writing the solution of equation (14) that satisfies conditions (7) for $0 \le x \le 1$, (10), (12) and differentiating this solution with respect to x and tending x to zero and unity, after lengthy calculations we obtain a system of Volterra integral equations of the second kind with respect to $\tau_4'(y)$ and $\tau_5'(y)$. Solving this system, we find the functions $\tau_4'(y)$, $\tau_5'(y)$ and thus the functions $\nu_4(y)$, $\nu_5(y)$, $u_1(x,y)$, $u_3(x,y)$, $u_4(x,y)$.

Thus, we have found a solution to the problem in a unique way.

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