ON A NONLOCAL PROBLEM FOR THE EQUATION OF THE THIRD ORDER WITH MULTIPLE CHARACTERISTICS

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Annotation. ***The article studied the issue of nolocal for the third order equation of characteristic multiplicity.***

Keywords: ***boundary conditions, integral equations***

1. Introduction It is known that in the work of E. Del Vecchio gives a method for constructing fundamental solutions of an equation with multiple characteristics and as an application the fundamental solution of the equation is constructed (см.[1])

$$Lu \equiv \frac{\partial^3 u}{\partial x^3} - \frac{\partial^2 u}{\partial t^2} = 0. \qquad (1)$$

Further, L.Cattabriga developing the work of E.Del Vecchio in 1959 constructed the fundamental solutions of the equation (см.[2])

$$Lu \equiv \frac{\partial^{2n+1} u}{\partial x^{2n+1}} - (-1)^n \frac{\partial^2 u}{\partial t^2} = 0, \ n \in \mathrm{N}, n < \infty \qquad (2)$$

and developed the theory of potentials of fundamental solutions. Later, the researchers considered a number of boundary value problems for equation (1) with local boundary conditions, for example, (см.[2]-[4]).

In the area of $\Omega = \{(x,t) : 0 < x < 1,\ 0 < t < T\}$ consider the equation

$$Lu \equiv \frac{\partial^3 u}{\partial x^3} - \frac{\partial^2 u}{\partial t^2} = 0. \qquad (3)$$

with non-local boundary conditions

$$u(x,0) = u(x,T), \qquad u_t(x,0) = u_t(x,T), \qquad (4)$$

$$u_x(0,t) = \varphi(t), \qquad u(1,t) = \psi_1(t), \qquad u_x(1,t) = \psi_2(t). \qquad (5)$$

in the classroom $u(x,t) \in C^{3,2}_{x,t}(\Omega) \cap C^{2,1}_{x,t}(\overline{\Omega})$.

ISSN 2277-3630 (online), Published by International journal of Social Sciences & Interdisciplinary Research., under Volume: 11 Issue: 06 in June-2022
https://www.gejournal.net/index.php/IJSSIR

It is known that the fundamental solutions of equation (3) have the following form (см. [3]).

$$U(x-\xi;t-\tau)=|t-\tau|^{1/3}\, f\left(\frac{x-\xi}{|t-\tau|^{2/3}}\right),\;\; x\neq\xi,\; t\neq\tau; \tag{6}$$

$$V(x-\xi;t-\tau)=|t-\tau|^{1/3}\, \varphi\left(\frac{x-\xi}{|t-\tau|^{2/3}}\right),\;\; x<\xi,\; t\neq\tau. \tag{7}$$

Here

$$f(z)=\frac{2}{3}|z|^{1/2}\int_{z}^{\infty}\eta^{-3/2} f^{*}(\eta)d\eta+c^{+},\quad z>0, \tag{8}$$

$$f(z)=\frac{2}{3}|z|^{1/2}\int_{-\infty}^{z}\eta^{-3/2} f^{*}(\eta)d\eta+c^{-},\quad z<0, \tag{9}$$

$$\varphi(z)=\frac{2}{3}|z|^{1/2}\int_{-\infty}^{z}\eta^{-3/2} \varphi^{*}(\eta)d\eta+c,\quad z<0, \tag{10}$$

$$f^{*}(z)=\int_{0}^{\infty}\exp\left(-\frac{\lambda^{3/2}}{\sqrt{2}}\right)\cos\left(\frac{\lambda^{3/2}}{\sqrt{2}}+\lambda z\right)d\lambda,\quad -\infty<z<\infty,$$

$$\varphi^{*}(z)=\int_{0}^{\infty}\exp\left(\lambda z-\lambda^{3/2}\right)d\lambda+\int_{0}^{\infty}\exp\left(-\frac{\lambda^{3/2}}{\sqrt{2}}\right)\sin\left(\frac{\lambda^{3/2}}{\sqrt{2}}+\lambda z\right)d\lambda,\quad z<0,$$

$$z=(x-\xi)|t-\tau|^{-2/3}.$$

For the function

$$U(x-\xi;t-\tau),\qquad V(x-\xi;t-\tau),$$
$$U^{*}(x-\xi;t-\tau),\qquad V^{*}(x-\xi;t-\tau),$$
$$f(z),\quad \varphi(z),\quad f^{*}(z),\quad \varphi^{*}(z)$$

the ratios are fair

$$f''(z)+\frac{2}{3}zf^{*}(z)=0,\qquad \varphi''(z)+\frac{2}{3}z\varphi^{*}(z)=0, \tag{11}$$

$$\int_{-\infty}^{\infty} f^{*}(z)=\pi,\qquad \int_{-\infty}^{0} f^{*}(z)=\frac{2\pi}{3},\qquad \int_{0}^{\infty} f^{*}(z)=\frac{\pi}{3},\qquad \int_{-\infty}^{0}\varphi^{*}(z)=0, \tag{12}$$

ISSN 2277-3630 (online), Published by International journal of Social Sciences & Interdisciplinary Research., under Volume: 11 Issue: 06 in June-2022
https://www.gejournal.net/index.php/IJSSIR

$$U_t = -U_\tau = sign(t-\tau)U^*, \qquad V_t = -V_\tau = sign(t-\tau)V^*, \tag{13}$$

$$\lim_{\tau\to\pm t}\int_a^b U^*(x-\xi;t-\tau)\alpha(\xi,\tau)d\xi = \pm\pi\alpha(x,t), \quad x\in[a,b], \tag{14}$$

$$\lim_{\tau\to t}\int_a^b U^*(x-\xi;t-\tau)\alpha(\xi,\tau)d\xi = 0, \quad x\overline{\in}[a,b]. \tag{15}$$

$$\lim_{\xi\to+0}\int_\tau^t U_{\xi\xi}(0-\xi;t-\tau)\alpha(\xi,\tau)d\tau = \frac{2\pi}{3}\alpha(t), \tag{16}$$

$$\lim_{\xi\to-0}\int_\tau^t U_{\xi\xi}(0-\xi;t-\tau)\alpha(\xi,\tau)d\tau = -\frac{\pi}{3}\alpha(t), \tag{17}$$

$$\lim_{\xi\to+0}\int_\tau^t V_{\xi\xi}(0-\xi;t-\tau)\alpha(\xi,\tau)d\tau = 0, \tag{18}$$

$$\left|\frac{\partial^{h+k}U}{\partial x^h\partial t^k}\right| < \frac{|x-\xi|^{-\frac{2h+3k+\frac{1}{2}-(-1)^k}{2}}}{|t-\tau|^{\frac{1-(-1)k}{2}}}, \qquad \frac{x-\xi}{|t-\tau|^{\frac{2}{3}}}\to-\infty, \tag{19}$$

$$\left|\frac{\partial^{h+k}V}{\partial x^h\partial t^k}\right| < \frac{|x-\xi|^{-\frac{2h+3k+\frac{1}{2}-(-1)^k}{2}}}{|t-\tau|^{\frac{1-(-1)k}{2}}}, \qquad \frac{x-\xi}{|t-\tau|^{\frac{2}{3}}}\to-\infty, \tag{20}$$

$$\left|\frac{\partial^{h+k}U}{\partial x^h\partial t^k}\right| < |t-\tau|^{-\frac{2h+3k-1}{3}}\exp\left(-\left(\frac{x-\xi}{|t-\tau|^{\frac{2}{3}}}\right)^3\right), \qquad \frac{x-\xi}{|t-\tau|^{\frac{2}{3}}}\to\infty, \tag{21}$$

where

$$U^*(x-\xi;t-\tau) = |t-\tau|^{-1/3} f^*\left(\frac{x-\xi}{|t-\tau|^{2/3}}\right), \quad x\neq\xi, \quad t\neq\tau, \tag{22}$$

$$V^*(x-\xi;t-\tau) = |t-\tau|^{-1/3} \varphi^*\left(\frac{x-\xi}{|t-\tau|^{2/3}}\right), \quad x<\xi, \quad t\neq\tau. \tag{23}$$

2. Main results

ISSN 2277-3630 (online), Published by International journal of Social Sciences & Interdisciplinary Research., under Volume: 11 Issue: 06 in June-2022
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Theorem 1. *Problem (3)-(5) does not have more than one solution.*

Proof. Let the problem (3)-(5) have two solutions: $u_1(x,t), u_2(x,t)$. Then assuming $v(x,t) = u_1(x,t) - u_2(x,t)$ we get a problem of type (3)-(5) with respect to the function $v(x,t)$ with homogeneous boundary conditions. Now consider the identity

$$\int_0^1\int_0^T L(v)v_x(x,t)dxdt = 0. \quad (24)$$

Integrating in parts, taking into account homogeneous boundary conditions of type (5), (6), we have

$$-\int_0^1\int_0^T v_{xx}^2(x,t)dxdt - \frac{1}{2}\int_0^1 v_t^2(0,t)dt = 0$$

From here, $v_{xx}(x,t) = 0$ в Ω, $v_t(0,t) = 0$ в $[0,T]$.

Since $v_{xx}(x,t) = 0$, то $v_x(x,t) = \lambda_1(t)$, $v(x,t) = x\lambda_1(t) + \lambda_2(t)$. By assumption function $v(x,t) = x\lambda_1(t) + \lambda_2(t)$ is a solution of problem (3)-(5) with homogeneous boundary conditions. Therefore

$$v(0,t) = \lambda_2(t), \quad v(1,t) = \lambda_1(t) + \lambda_2(t) = 0 \Rightarrow \lambda_1(t) = -\lambda_2(t)$$

On the other hand

$$v_t(0,t) = 0 \Rightarrow v(0,t) = const.$$

Then $\lambda_2(t) = const \Rightarrow \lambda_1(t) = -const$. By virtue of this

$$v(x,t) = (1-x)const \Rightarrow v_x(x,t) = -const.$$

Since $v_x(0,t) = 0$, $v_x(1,t) = 0$, то $const = 0$. Therefore $v(x,t) \equiv 0$ в $\overline{\Omega}$.

Theorem 2. *Let* $\psi_1(t) \in C^1([0,T])$, $\psi_2 \in C^1([0,T])$, $\varphi(t) \in C([0,T])$. *Then there is a solution to the problem* (3)-(5).

Proof. Consider two auxiliary tasks:

I. In the area of $\Omega = \{(x,t): 0 < x < 1, 0 < t < T\}$ consider the equation

$$Lu \equiv \frac{\partial^3 u}{\partial x^3} - \frac{\partial^2 u}{\partial t^2} = 0. \quad (25)$$

with boundary conditions

$$u(x,0) = u(x,T) = \alpha(x), \quad (26)$$

ISSN 2277-3630 (online), Published by International journal of Social Sciences & Interdisciplinary Research., under Volume: 11 Issue: 06 in June-2022
https://www.gejournal.net/index.php/IJSSIR

$$u_x(0,t)=\varphi(t),\quad u(1,t)=\psi_1(t),\quad u_t(1,t)=\psi_2(1,x),\tag{27}$$

где $u(x,0)=\alpha(x)\in C^3((0,1))\cap C^2([0,1])$ unknown function yet.

Due to the work of [4], the solution of the problem (25)-(27) will be in the following form

$$2\pi u(x,t)=\int_0^T G_{\xi\xi}(x-1;t-\tau)\psi_1(\tau)d\tau-$$

$$-\int_0^T G_\xi(x-1;t-\tau)\psi_2(\tau)d\tau+\int_0^T G_\xi(x-0;t-\tau)\varphi(\tau)d\tau+$$

$$+\int_0^1\{G_\tau(x-\xi;t-T)-G_\tau(x-\xi;t-0)\}\alpha(\xi)d\xi,\tag{28}$$

where

$$G(x-\xi;t-\tau)=U(x-\xi;t-\tau)-W(x-\xi;t-\tau),$$

function $W(x-\xi;t-\tau)$ is a solution to the following problem

$$M(W)\equiv-\frac{\partial^3 W}{\partial x^3}-\frac{\partial^2 W}{\partial t^2}=0,$$

$$U|_{\xi=1}=W|_{\xi=1},\quad U|_{\xi=0}=W|_{\xi=0},\quad U_{\xi\xi}|_{\xi=0}=W_{\xi\xi}|_{\xi=0},$$

$$U|_{\tau=0}=W|_{\tau=0},\quad U|_{\tau=T}=W|_{\tau=T}\ .$$

Now differentiate (28) by , then proceed to the limit $t\to 0$. Then denoting $\beta(x)=u_t(x,0)$ we get the relation between the functions $\alpha(x)$ и $\beta(x)$

$$2\pi\beta(x)=\int_0^T G_{\xi\xi}(x-1;0-\tau)\psi_1'(\tau)d\tau-$$

$$-\int_0^T G_\xi(x-1;0-\tau)\psi_2'(\tau)d\tau+\int_0^T G_\xi(x-0;0-\tau)\varphi'(\tau)d\tau+$$

$$+\int_0^1 G_\xi(x-\xi;0-T)\alpha''(\xi)d\xi.\tag{29}$$

II. In the area of $\Omega=\{(x,t):0<x<1,\ 0<t<T\}$ consider the equation

ISSN 2277-3630 (online), Published by International journal of Social Sciences & Interdisciplinary Research., under Volume: 11 Issue: 06 in June-2022
https://www.gejournal.net/index.php/IJSSIR

$$Lu \equiv \frac{\partial^3 u}{\partial x^3} - \frac{\partial^2 u}{\partial t^2} = 0. \tag{30}$$

with boundary conditions

$$u_t(x,0) = u_t(x,T) = \beta(x), \tag{31}$$

$$u_x(0,t) = \varphi(t), \quad u(1,t) = \psi_1(t), \quad u_t(1,t) = \psi_2(1,x), \tag{32}$$

where $u_t(x,0) = \beta(x) \in C^2((0,1)) \cap C^1([0,1])$ unknown function yet.

Due to the work [4], the solution of the problem (29)-(31) will be in the following form

$$2\pi u(x,t) = \int_0^T G_{\xi\xi}(x-1;t-\tau)\psi_1(\tau)d\tau -$$

$$-\int_0^T G_{\xi}(x-1;t-\tau)\psi_2(\tau)d\tau + \int_0^T G_{\xi}(x-0;t-\tau)\varphi(\tau)d\tau +$$

$$+\int_0^1 \{G(x-\xi;t-0) - G(x-\xi;t-T)\}\beta(\xi)d\xi, \tag{33}$$

where

$$G(x-\xi;t-\tau) = U(x-\xi;t-\tau) - W(x-\xi;t-\tau),$$

function $W(x-\xi;t-\tau)$ is a solution to the following problem

$$M(W) \equiv -\frac{\partial^3 W}{\partial x^3} - \frac{\partial^2 W}{\partial t} = 0,$$

$$U|_{\xi=1} = W|_{\xi=1}, \quad U|_{\xi=0} = W|_{\xi=0}, \quad U_{\xi\xi}|_{\xi=0} = W_{\xi\xi}|_{\xi=0},$$

$$U_\tau|_{\tau=0} = W_\tau|_{\tau=0}, \quad U_\tau|_{\tau=T} = W_\tau|_{\tau=T}.$$

Now moving to the limit of (33) we get the second relation between the functions $\alpha(x)$ и $\beta(x)$

$$2\pi\alpha(x) = \int_0^T G_{\xi\xi}(x-1;0-\tau)\psi_1(\tau)d\tau -$$

$$-\int_0^T G_{\xi}(x-1;0-\tau)\psi_2(\tau)d\tau + \int_0^T G_{\xi}(x-0;0-\tau)\varphi(\tau)d\tau -$$

ISSN 2277-3630 (online), Published by International journal of Social Sciences & Interdisciplinary Research., under Volume: 11 Issue: 06 in June-2022
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$$-\int_0^1 G(x-\xi;0-T)\beta(\xi)d\xi. \tag{34}$$

So we have obtained a system of integral equations (29), (34) with respect to functions $\alpha''(x)$ и $\beta(x)$.

We exclude the system from this $\alpha''(x)$ and we get an integral Fredholm - type equation with respect to the function $\beta(x)$

$$\beta(x)=\int_0^1 K(x,\xi)\beta(\xi)d\xi+F(x), \tag{35}$$

где $|K(x,\xi)|<\dfrac{C}{|x-\xi|^{1/2}},\ F(x)\in C^1([0,1]).$

Due to the uniqueness of the solution of the problem (3)-(6), the integral equation (35) has a unique solution.

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ISSN 2277-3630 (online), Published by International journal of Social Sciences & Interdisciplinary Research., under Volume: 11 Issue: 06 in June-2022
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