ABOUT ONE PROBLEM FOR THE EQUATION OF THE THIRD ORDER WITH A NON - LOCAL CONDITION

Turgunov Azizjon Mamsliyevich

Teacher of the Kokand State Pedagogical Institute

Asqaraliyeva Muxtasarxon Azizjon qizi

Teacher of the Kokand State Pedagogical Institute

Annotation. ***In the paper nonlocal problem for equation of the third order with multiple characteristics are consider***

Keywords: ***boundary conditions, integral equations***

1. Introduction. It is known that in the work of E.Del Vecchio gives a method for constructing fundamental solutions of an equation with multiple characteristics and as an application, a fundamental solution of the equations is constructed (см.[1])

$$Lu \equiv \frac{\partial^3 u}{\partial x^3} - \frac{\partial u}{\partial t} = 0, \tag{1}$$

$$Lu \equiv \frac{\partial^3 u}{\partial x^3} - \frac{\partial^2 u}{\partial t^2} = 0. \tag{2}$$

Further, L.Cattabriga developing the work of E.Del Vecchio in 1961 investigated the properties of the potentials of the fundamental solutions of equation (1), i.e. he built the theories of the potentials of the fundamental solutions of the equation (see [2]). Later, the researchers considered a number of boundary value problems for equation (1) with local and non-local boundary conditions, for example, (см.[2]-[5]).

In this paper, the following problem is considered:

Need to find a function $u(x,t) \in K_u$, which is a regular solution of the equation

$$Lu \equiv \frac{\partial^3 u}{\partial x^3} - \frac{\partial u}{\partial t} = 0. \tag{3}$$

in the area of $\Omega = \{(x,t): 0 < x < 1,\ 0 < t \le T\}$ and satisfies the conditions

$$u(x,0) = \mu u(x,T), \qquad \mu = const, \tag{4}$$

$$u(0,t) = \varphi_1(t), \quad u_x(0,t) = \varphi_2(t), \quad u_x(1,t) = \psi(t). \tag{5}$$

Here $K_u = \{u(x,t): u(x,t) \in C^{3,1}_{x,t}(\Omega) \cap C^{2,0}_{x,t}(\overline{\Omega}),\ u_{xt} \in C(\Omega)\}$.

It is known that the fundamental solutions of equation (2) have the form (см. [2]).

ISSN 2277-3630 (online), Published by International journal of Social Sciences & Interdisciplinary Research., under Volume: 11 Issue: 06 in June-2022
https://www.gejournal.net/index.php/IJSSIR

$$U(x-\xi;t-\tau)=(t-\tau)^{-1/3}f\left(\frac{x-\xi}{(t-\tau)^{1/3}}\right),\quad x\neq\xi,\quad t>\tau; \tag{6}$$

$$V(x-\xi;t-\tau)=(t-\tau)^{-1/3}\varphi\left(\frac{x-\xi}{(t-\tau)^{1/3}}\right),\quad x>\xi,\quad t>\tau. \tag{7}$$

Here

$$f(z)=\int_0^\infty\cos\left(\lambda^3-\lambda z\right)d\lambda,-\infty<z<\infty,$$

$$\varphi(z)=\int_0^\infty(\exp\left(-\lambda^3-\lambda z\right)+\sin\left(\lambda^3-\lambda z\right))d\lambda,\ z>0,$$

$$z=(x-\xi)(t-\tau)^{-1/3}.$$

For the function $U(x-\xi;t-\tau)$, $V(x-\xi;t-\tau)$, $f(z)$, $\varphi(z)$ the following relations are valid

$$f''(z)+\frac{1}{3}zf(z)=0,\qquad \varphi''(z)+\frac{1}{3}z\varphi(z)=0, \tag{8}$$

$$\int_{-\infty}^{\infty}f(z)=\pi,\quad \int_{-\infty}^{0}f(z)=\frac{\pi}{3},\quad \int_0^\infty f(z)=\frac{2\pi}{3},\quad \int_0^\infty\varphi(z)=0, \tag{9}$$

$$\lim_{(x,t)\to(a-0,t)}\int_\tau^t U_{\xi\xi}(x-a;t-\tau)\alpha(\xi,\tau)d\tau=\frac{\pi}{3}\alpha(t), \tag{10}$$

$$\lim_{(x,t)\to(a+0,t)}\int_\tau^t U_{\xi\xi}(x-a;t-\tau)\alpha(\xi,\tau)d\tau=-\frac{2\pi}{3}\alpha(t), \tag{11}$$

$$\lim_{(x,t)\to(a+0,t)}\int_\tau^t V_{\xi\xi}(x-a;t-\tau)\alpha(\xi,\tau)d\tau=0, \tag{12}$$

$$f^n(z):\ c_n^+ z^{\frac{2n-1}{4}}\sin\left(\frac{2}{3}z^{3/2}\right),\quad z\to\infty, \tag{13}$$

$$\varphi^n(z):\ c_n^+ z^{\frac{2n-1}{4}}\sin\left(\frac{2}{3}z^{3/2}\right),\quad z\to\infty, \tag{14}$$

ISSN 2277-3630 (online), Published by International journal of Social Sciences & Interdisciplinary Research., under Volume: 11 Issue: 06 in June-2022
https://www.gejournal.net/index.php/IJSSIR

$$f^n(z) \sim c_n^- |z|^{\frac{2n-1}{4}} \exp\left(-\frac{2}{3}|z|^{3/2}\right), \quad z \to -\infty, \tag{15}$$

2. Main results

The theorem 1. *Let* $\mu^2 \le \exp\{-T\}$. *Then problem (3)-(5) does not have more than one solution.*

Proof. Let the problem (3)-(5) have two solutions: $u_1(x,t)$, $u_2(x,t)$. Then assuming $v(x,t) = u_1(x,t) - u_2(x,t)$ we get the following problem with respect to the function $v(x,t)$

$$Lv \equiv \frac{\partial^3 v}{\partial x^3} - \frac{\partial v}{\partial t} = 0. \tag{16}$$

$$v(x,0) = \mu v(x,T), \tag{17}$$

$$v(0,t) = 0, \quad v_x(0,t) = 0, \quad v_x(1,t) = 0. \tag{18}$$

Consider the identity

$$\int_0^1\int_0^T L(v) v_{xt} \exp\{-t\} dxdt = 0. \tag{19}$$

Integrating in parts, taking into account homogeneous boundary conditions (17), (18), we have

$$-\frac{1}{2}\int_0^1\int_0^T v_{xt}^2(x,t)\exp\{-t\}dxdt - \frac{1}{2}\int_0^T v_x^2(1,t)\exp\{-t\}dt -$$

$$-\frac{1}{2}\int_0^1 v_{xx}^2(x,T)\{\exp\{-T\} - \mu^2\}dx = 0$$

From here, $v_{xx}(x,t) = 0$ в Ω, $v_{xx}(x,T) = 0$ в $x \in [0,1]$, $v_t(1,t) = 0$ в $t \in [0,T]$.

Let $\mu^2 < \exp\{-T\}$. Then from these inputs we get: $v_{xx}(x,T) = 0 \Rightarrow v_x(x,T) = const$.

Since $v_x(0,t) = v_x(1,t) = 0 \Rightarrow v_x(0,0) = v_x(0,T) = 0$, то $v_x(x,T) = const = 0$ by $\forall x \in [0,1]$.

Next, we have $v_x(x,T) = 0 \Rightarrow v(x,T) = const \Rightarrow v(x,0) = const$. Since $v(0,t) = 0 \Rightarrow v(0,0) = 0$, то $v(x,0) = const = 0$ by $\forall x \in [0,1]$. Due to the fact that $v_t(1,t) = 0 \Rightarrow v(1,t) = const$ and $v(0,0) = 0$ we have $v(1,t) = 0$.

Then we get the following boundary value problem with respect to the function $v(x,t)$

ISSN 2277-3630 (online), Published by International journal of Social Sciences & Interdisciplinary Research., under Volume: 11 Issue: 06 in June-2022
https://www.gejournal.net/index.php/IJSSIR

$$Lv \equiv \frac{\partial^3 v}{\partial x^3} - \frac{\partial v}{\partial t} = 0.$$

$$v(x,0) = 0, \quad v(0,t) = 0, \quad v_x(0,t) = 0, \quad v(1,t) = 0.$$

Due to the work [2], this problem has a unique solution.

Now let $\mu^2 = \exp\{-T\}$. Then $v_{xx} = 0 \Rightarrow v_x(x,t) = \delta_1(t)$ by $\forall t \in [0,T]$. Since $v_x(0,t) = v_x(1,t) = 0$, by $\forall t \in [0,T]$, that $\delta_1(t) = 0$ by $\forall t \in [0,T]$.

Further, $v_x = 0 \Rightarrow v(x,t) = \delta_2(t)$ by $\forall t \in [0,T]$. Since $v(0,t) = 0$, by $\forall t \in [0,T]$, that $\delta_2(t) = 0$ by $\forall t \in [0,T]$.

Then by virtue of continuity $v(x,t) = 0$ в $\overline{\Omega}$.

The theorem 2. *Let* $\psi(t) \in C^1([0,T]), \quad \varphi_2(t) \in C^1([0,T]), \quad \varphi_1(t) \in C^2([0,T])$. *Then there is a solution to the problem* (3)-(5).

Proof. Consider an auxiliary problem:

Find a function $u(x,t) \in K_u$, which is a regular solution of the equation

$$Lu \equiv \frac{\partial^3 u}{\partial x^3} - \frac{\partial u}{\partial t} = 0. \tag{20}$$

in the area of $\Omega = \{(x,t): 0 < x < 1,\ 0 < t \le T\}$ and satisfies the conditions

$$u(x,0) = \tau(x), \tag{21}$$

$$u(0,t) = \varphi_1(t), \quad u_x(0,t) = \varphi_2(t), \quad u(1,t) = \psi(t). \tag{22}$$

Due to the work [3], the solution of the problem (23)-(25) will be in the following form

$$\pi u(x,t) = -\int_0^t G_\xi(x-1;t-\tau)\psi(\tau)d\tau -$$

$$-\int_0^t G_{\xi\xi}(x-0;t-\tau)\varphi_1(\tau)d\tau + \int_0^T G_\xi(x-0;t-\tau)\varphi_2(\tau)d\tau +$$

$$+\int_0^1 G(x-\xi;t-0)\tau(\xi)d\xi, \tag{23}$$

where

$$G(x-\xi;t-\tau) = U(x-\xi;t-\tau) - W(x-\xi;t-\tau),$$

ISSN 2277-3630 (online), Published by International journal of Social Sciences & Interdisciplinary Research., under Volume: 11 Issue: 06 in June-2022
https://www.gejournal.net/index.php/IJSSIR

function $W(x-\xi;t-\tau)$ is a solution to the following problem

$$M(W) \equiv -\frac{\partial^3 W}{\partial x^3} - \frac{\partial W}{\partial t} = 0,$$

$$U|_{\xi=1} = W|_{\xi=1}, \quad U_{\xi\xi}|_{\xi=1} = W_{\xi\xi}|_{\xi=1}, \quad U|_{\xi=0} = W|_{\xi=0},$$

$$W|_{\tau=t} = 0.$$

Denote $u(x,T) = \alpha(x)$. Then going to the limit $t \to T$ from (26) we get

$$\pi\alpha(x) = -\int_0^t G_\xi(x-1;T-\tau)\psi(\tau)d\tau -$$

$$-\int_0^t G_{\xi\xi}(x-0;T-\tau)\varphi_1(\tau)d\tau + \int_0^T G_\xi(x-0;T-\tau)\varphi_2(\tau)d\tau +$$

$$+\mu\int_0^1 \{G(x-\xi;T-0)\alpha(\xi)d\xi, \qquad (24)$$

So we have obtained an integral Fredholm type equation with respect to the function $\alpha(x)$

$$\alpha(x) = \int_0^1 K(x,\xi)\alpha(\xi)d\xi + F(x), \qquad (25)$$

where

$$\mu G(x-\xi;T-0) \equiv |K(x,\xi)| < \frac{C}{|x-\xi|^{1/4}},$$

$$-\int_0^t G_\xi(x-1;T-\tau)\psi(\tau)d\tau - \int_0^t G_{\xi\xi}(x-0;T-\tau)\varphi_1(\tau)d\tau +$$

$$+\int_0^T G_\xi(x-0;T-\tau)\varphi_2(\tau)d\tau \equiv F(x) \in C^3([0,1]).$$

By virtue of the uniqueness of the solution of the problem (3)-(5), the integral equation (25) has a unique solution.

REFERENCES

1. *Del Vecchio E.* Sulle equazioni $z_{xxx} - z_y + \varphi_1(x,y) = 0$, $z_{xxx} - z_{yy} + \varphi_2(x,y) = 0$. Memorie R. Accad. Sci. Torino (2). 66 (1915).
2. *Cattabriga L.* Un problema al per una equazione parabolica di ordine dispari. Annali della Souola

ISSN 2277-3630 (online), Published by International journal of Social Sciences & Interdisciplinary Research., under Volume: 11 Issue: 06 in June-2022
https://www.gejournal.net/index.php/IJSSIR

Normale Sup. di Pisa a mat. 1959, Vol. 13, 2, p. 163-203.
3. *Абдиназаров С.* Об одном уравнения третьего порядка. Изв.АН УзССР, Сер.физ.мат. наук 1989, 6, с 3-7.
4. *Хашимов А.* Об одной задаче для уравнения смешанного типа с кратными характеристиками. Уз. Мат. Журнал, 1995, 2, с 95-97.
5. Ravshanbek, J. (2022). CREDIT-MODULE SYSTEM, ITS BASIC PRINCIPLES AND FEATURES. Yosh Tadqiqotchi Jurnali, 1(4), 304-309.
6. O'G'Li, J. R. M. (2022). METHODS OF ORGANIZING INDEPENDENT STUDY OF STUDENTS IN THE CREDIT-MODULE SYSTEM. Ta'lim fidoyilari, 25(5), 93-97.
7. Jorayev, N. S. (2021, July). QUALITY LEARNING PROCESS-AS A MECHANICAL SUM OF TEACHING AND LEARNING PROCESSES. In Euro-Asia Conferences (pp. 57-59).
8. Sadullayevich, J. N. (2021). Improving psychological technologies for the development of professional reflection in future teachers. ACADEMICIA: AN INTERNATIONAL MULTIDISCIPLINARY RESEARCH JOURNAL, 11(1), 229-232.
9. Джураева, M. A. (2022, May). ПРИЁМЫ КОНТРОЛЯ УРОВНЯ ЗНАНИЙ НА УРОКАХ РУССКОГО ЯЗЫКА И ЛИТЕРАТУРЫ. In INTERNATIONAL SCIENTIFIC RESEARCH CONFERENCE (Vol. 1, No. 3, pp. 28-32).
10. Abdukakhhorovna, Z. M. (2022). Lexical Polysemy of the Russian Language. Middle European Scientific Bulletin, 22, 77-81.
11. Устаджалилова, Х., Хайдарова, М., & Олимова, Д. (2020). РОЛЬ ИСТОРИЧЕСКОГО И КУЛЬТУРНОГО НАСЛЕДИЯ В ФОРМИРОВАНИИ МОТИВАЦИИ ИЗУЧЕНИЯ МАТЕМАТИКИ. In Фундаментальные и прикладные научные исследования: актуальные вопросы, достижения и инновации (pp. 17-19).
12. Султанов, Д., & Устаджалилова, Х. А. (2014). Особенности развития геометрических умений и навыков учащихся при решении задач методом геометрических преобразований. In Теория и практика современных гуманитарных и естественных наук (pp. 253-259).
13. Mamayusupova, I. K. (2020). ON THE PSYCHOLOGICAL CRITERIA AND FACTORS OF ORIGIN OF CONFLICTS THAT ARISE BETWEEN YOUNG PEOPLE. Theoretical & Applied Science, (2), 630-633.
14. Khamidovna, M. I. (2021). Different ways of Resolving and Managing Conflicts. Middle European Scientific Bulletin, 17, 204-207.
15. Khamidovna, M. I., & Khudayberganov, O. (2022). The Psychology of Adolescent Conflicts in Society. Yosh Tadqiqotchi Jurnali, 1(1), 29-33.
16. Khamidovna, M. I. (2021). Organizational and socio-psychological mechanisms for making managerial decisions in conflict situations between adolescents.
17. Muminova, G. B. Senior scientific researcher, TSEU Social Media as a tool of Innovative Marketing: The case of Uzbekistan.
18. Muqimovna, G. A. D., & Baxodirovna, M. G. (2022). TECHNOLOGY FOR THE FORMATION OF SOCIAL CONSCIOUSNESS IN PRESCHOOL ADULTS. INTERNATIONAL JOURNAL OF SOCIAL SCIENCE & INTERDISCIPLINARY RESEARCH ISSN: 2277-3630 Impact factor: 7.429, 11(05), 221-227.

ISSN 2277-3630 (online), Published by International journal of Social Sciences & Interdisciplinary Research., under Volume: 11 Issue: 06 in June-2022
https://www.gejournal.net/index.php/IJSSIR